Analysis of Oil and Gas Production Performance

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Society of Petroleum Engineers

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Dedication

This book is dedicated to the past, present, and future petroleum-engineering students passing through Texas A&M University.

Additionally, R. L. Whiting and D. von Gonten, former heads of the Department of Petroleum Engineering at Texas A&M University, should always be remembered for their foresight and efforts to expand and extend the scope of the department to the parents, state, and industry. R. Berg and R. Morse were of a great guidance and help while we were at the department.

Preface

Oil and gas companies are continually seeking and applying new technologies, processes, and methods to reduce their cost of finding and producing hydrocarbons while remaining competitive in the current and changing global economy. Improving the efficiency of business processes and maximizing the productivity of the workforce will help to reduce the associated costs and should ultimately increase profitability.

Although technology has helped companies to better evaluate the prospects, lack of trained geoscientists and engineers and the absence of proper vehicles for training and technology transfer may jeopardize oil- and gasfield-development efforts. An efficient and effective way to help develop core competencies for different jobs is to design tools and training actions to address these needs. In this book, workflows have been developed that apply key technology independently by analyzing the processes and solving example problems, thereby addressing the importance of integration of subsurface disciplines related to oil and gas exploration and production.

Traditional Arp's models exist that are based on graphical extrapolation of production data, and they have been regarded in our industry as one of the preferred and commonly-used tools for estimating future performance in oil and gas wells. However, the practical aspects of analyzing production performance have changed as a result of the increased exploitation efforts in unconventional reservoirs. The complexities of these types of reservoirs were not adequately covered in the initial work *Analysis of Production Decline Curves*, published by the Society of Petroleum Engineers in October 2008. In the current book, the scope has been broadened, and we provide many more field examples, including problems that cover the specific subjects of developing well-evaluation procedures and best practices for new areas of shale and tight formation reservoirs.

Advances in horizontal well drilling and multistage hydraulic fracturing have allowed industry to develop unconventional nano-Darcy permeability reservoirs (shale oil and shale gas). These highly-heterogeneous multiphase systems do not lend themselves to typical analytical solutions to predict future performance. Boundary conditions applied to such systems are based on ideal geometrical configurations and idealized flow theory. This approach implies important and sometimes faulty assumptions concerning geological heterogeneity and multiphase flow in the physical system. Aspects of production forecasting in unconventional resources are now covered in this book. The sections discussing type curve and two phase flow have been expanded and revised completely, and an additional section on types of equations replicating different flow conditions encountered in the oil field is presented. The most useful plotting and interpretive methods have been added, and a method for estimating ultimate recovery is included.

This book is intended for engineers, geologists, and anyone working in the oil and gas industry with an interest in production forecasting of conventional and unconventional resources for evaluation and development. The majority of the book is concerned with commonly observed oilfield practice and practical solutions to the problems encountered therein. Each chapter begins with a workflow diagram that, in essence, provides the reader with the learning objectives of the chapter. A primary focus of the book is to instill each reader with the competency to solve typical operational problems with minimal exposure to the complexity of the underlying mathematics and equations. The basics and utility of each equation are discussed; however, the focus is on the practical application of the underlying technology to real-life problems. There are numerous illustrations and solutions to typical field problems included for the reader.

About the Authors

Steven W. Poston is a retired professor emeritus from Texas A&M University, with more than 18 years of teaching experience at the graduate and undergraduate levels in applied reservoir engineering analysis and subsurface description of petroleum reservoirs both in the United States and internationally. His industry experience includes more than 14 years in a variety of subsurface engineering, geological, and managerial roles for Gulf Oil Exploration and Production Company in Nigeria, Pennsylvania, Louisiana, and Texas. Poston has coauthored numerous industry technical papers, has served on a number of SPE committees, and was coauthor of the SPE books *Overpressured Gas Reservoirs* and *Analysis of Production Decline Curves*.

Marcelo Laprea-Bigott is professor of engineering practice at the Harold Vance Department of Petroleum Engineering at Texas A&M University and has more than 45 years of consulting experience and an extensive background in developing and delivering technical training courses in the field of reservoir engineering for a variety of national oil companies and international clients. He joined Texas A&M University in 2006 after 20 years with Schlumberger Data and Consulting Services, having been an advisor in their Network of Excellence in Training (NExT). Laprea-Bigott's other private sector industry experience includes more than 13 years as founder, principal owner, and president of Simupet, C.A., a petroleum engineering consulting firm; Consorcio Lamar C.A., an integrated oilfield tubulars management company; and president of S. A. Holditch and Associates–Venezuela. Additionally, he was a professor at Universidad de Oriente–Venezuela, a visiting adjunct professor at Texas A&M University, a visiting adjunct professor at the University of Tulsa, and the former assistant director of the Energy Institute at Texas A&M University. Laprea-Bigott holds a PhD degree from Texas A&M University.

Bobby D. Poe Jr. is a production and reservoir engineer with more than 30 years of industrial experience in all levels of well stimulation, well performance, and reservoir-engineering analyses. His most recent engineering position is as the production interpretation advisor at Schlumberger in Houston. Poe holds a PhD degree in petroleum engineering from Texas A&M University. He has served on a number of SPE Reprint Series and editorial committees, has authored or coauthored numerous technical articles related to production analyses, and coauthored the SPE book *Analysis of Production Decline Curves*. Poe holds more than a dozen US and international patents related to production-engineering analyses and interpretation techniques.

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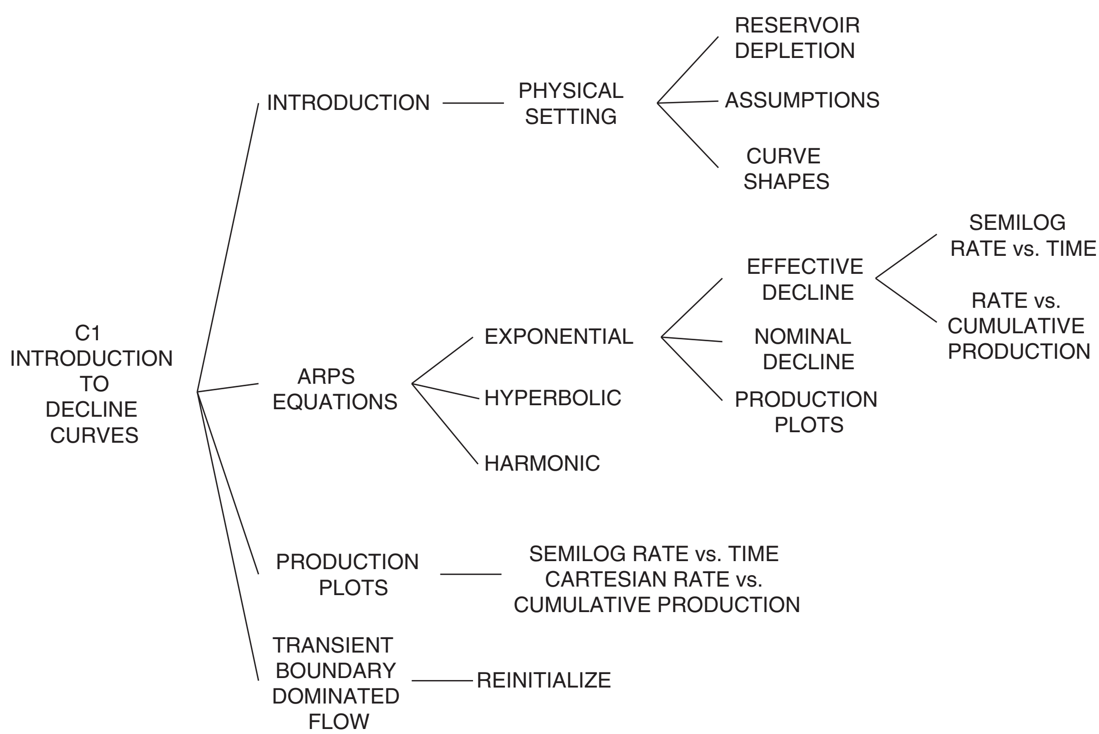
Chapter 1

Introduction to Decline Curves



The first chapter sets the foundation for the ensuing work, which delves deeply into different aspects of production decline analysis. The study of production performance is often denigrated because of the often-uncertain data quality. However, it is the one set of data most often available for estimating well character.

Every well or field does not lend itself to decline curve analysis. The reader is initially introduced to some of these general uncertainties and assumptions. One should be aware of these fundamentals before analyzing and predicting performance no matter how sophisticated the approach is.

Mathematically fitting an equation of a line to a production decline curve has been attempted by various authors in the past. However, Arps (1945) was the first to present a unified approach for analyzing a performance curve. Because of the complexity of the analysis process, estimating future performance with the Arps hyperbolic equation was not widely pursued until the advent of personal computers.

We will review these fundamentals in this chapter.

Introduction to Decline Curves

Production decline curve analysis is a classical reservoir engineering technique, applicable to both oil and gas wells. Production decline analysis is a traditional means of identifying well production problems and predicting well performance and well life on the basis of real production data. The decline curve analysis predictions are valid only if factors that influenced the performance trends of wells or fields in the past would continue to govern their performance in the same manner.

Oil and gas production rates generally decline as a function of time. Chief factors in the decline are discussed next.

Fitting a line through declining production values and assuming this same line trends forward in a similar manner forms the basis for analyzing decline curves. However, similarity of current and future performance is not necessarily a function of the equation of a line. In fact, the character production curve is derived from of the rock fabric, fluid type, completion characteristics and producing rate.

It has some important and generic applications:

- Can be conducted on well, reservoir, and field level
- Can be used to determine the reserves for a well, lease, or field
- Independent method of reserves estimation, the result of which on conventional reservoirs can be compared with volumetric or material-balance estimates
- Can be performed to estimate a base line to evaluate the success of future production enhancement (i.e. Future infill drilling, fluid injection, fracturing, acidizing) operations
- Can be used for the evaluation of new investments; audit of previous expenditures; sizing of equipment and facilities such as pipelines, plants, and treating facilities

Arps (1945) introduced the first systematic approach for the analysis of decline curves by empirical methods. Fetkovich (1980) introduced type curves and methodology to analyze transient and boundary-dominated flow periods, and Blasingame et al. (1991) and Agarwal et al. (1998) published work for using type curves and derivative curves accounting for flowing pressure variations.

Later work concentrated on the application of production decline analysis to fractured unconventional oil and gas systems. The classical Arp's approach uses empirical models with little fundamental justification and uses only production data, (no special reservoir parameters are required) and gives

- Forecast of future production rates
- Reserves estimation

Modern techniques involve a theoretical approach and account for pressure variations and reservoir parameters. Advanced decline curve analysis gives

- Estimation of k and S
- Distinction between transient and boundary flow
- Forecast of future production rates
- Reserves and original-oil-in-place (OOIP) and original-gas-in-place (OGIP) estimations

Chief factors for the oil and gas production rates decline as a function of time are

- Reservoir pressure provides energy to drive fluids from the reservoir (p_{res}) to the perforations (p_{wf}), and then to the surface (p_{tf}). Continued depletion of oil or gas fluids causes loss of reservoir pressure, which in turn affects production rate.
- Changing relative volumes of produced fluids. An unwanted fluid, such as water or gas in the case of an oil well or water in the case of a gas well, enters the flow stream. Decreased production of the primary product is the result of the onset of two-phase production and increased hydrostatic head.

Other frequent possible factors are

- Increase in near-wellbore damage (Skin>0)
- Production problems (e.g., sand production, scale, asphaltenes)

Fitting a line through declining production values and assuming this same line trends forward in a similar manner forms the basis for analyzing decline curves. However, similarity of current and future performance is not necessarily a function of the equation of a line. In fact, the character of the production curve is derived from of the rock fabric, fluid type, completion characteristics, and producing rate.

Physical Considerations

Production rates initially are dependent on growth of the expanding drainage system. Depletion is a function of an apparent increasing drainage volume (infinite-acting flow behavior also known as transient flow). On the other hand, encountering a reservoir boundary implies production is controlled by the drainage volume (boundary dominated flow). Including effects of infinite-acting flow implies an increasing reserves estimate. This fact presents a particular problem when studying very-low-permeability reservoirs.

Rocks are seldom distributed in a homogeneous manner but are often layered during the sedimentary process. Each layer is composed of rocks of different properties and furnishes different depletion rates to the flow stream. The expansion rate of a disturbance migrating outward from the wells is based on the diffusion constant $\left(\eta = \frac{k}{\phi \mu c_t}\right)$, where k = permeability, ϕ = porosity, μ = viscosity, and c_t = total compressibility.

One can see that a thousandfold difference in permeability could materially affect production rates from a low-permeability or layered sand. Including production derived from natural or hydraulic fractures would add further complexity of analysis because of their dual permeability.

Reservoir Depletion. Fig. 1.1 illustrates expansion of reservoir drainage limits from inception when the well is placed on production until an outer boundary (r_e) is encountered. The well is operating under constant flowing bottomhole-pressure (p_{wf}) conditions.

Boundary-Dominated Flow. The equation for calculating time required for a reservoir to transition from infinite-acting to boundary dominated flow conditions is:

$$t_{pss} \approx \frac{40 \phi \mu c_t r_e^2}{k} \quad (1.1)$$

where t_{pss} is in days.

Table 1.1 shows time required for a pressure disturbance to travel from the wellbore to the outer boundary for three reservoir cases. These calculations show boundary dominated flow can be initiated in a matter of a few days

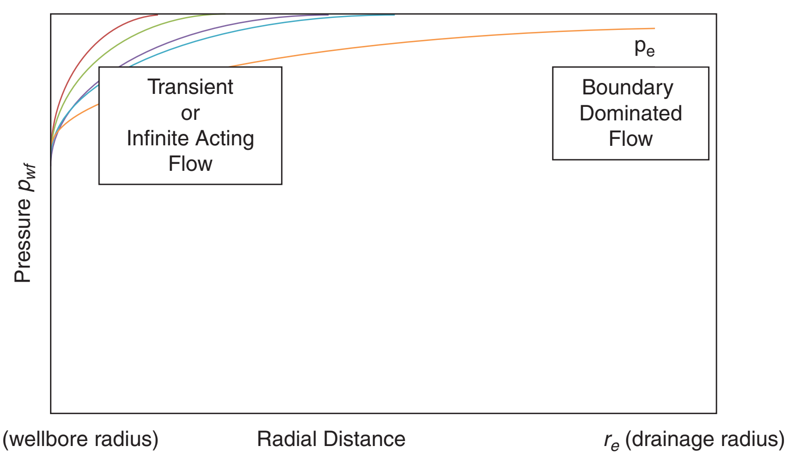


Fig. 1.1—Differentiating between constantly expanding (transient) and constant-volume (boundary dominated) conditions. An expanding drainage radius indicates an increasing reservoir volume. The pressure drop between (r_w) and (r_e) must begin to decline because a closed outer boundary has been encountered at boundary dominated conditions.

	Oil Reservoir	High-Pressure Gas Reservoir	Low-Pressure Gas Reservoir
Drainage area (acres)	160	640	640
Drainage area (sq ft)	1,490	2,980	2,980
Viscosity (cp)	0.6	0.022	0.018
Porosity (%)	12	12	12
Compressibility, 1/psi	20×10^{-6}	40×10^{-6}	170×10^{-6}
Permeability, md	50	10	100
t_{pss}, days	2.6	3.8	1.3

Table 1.1—Onset comparison of boundary conditions.

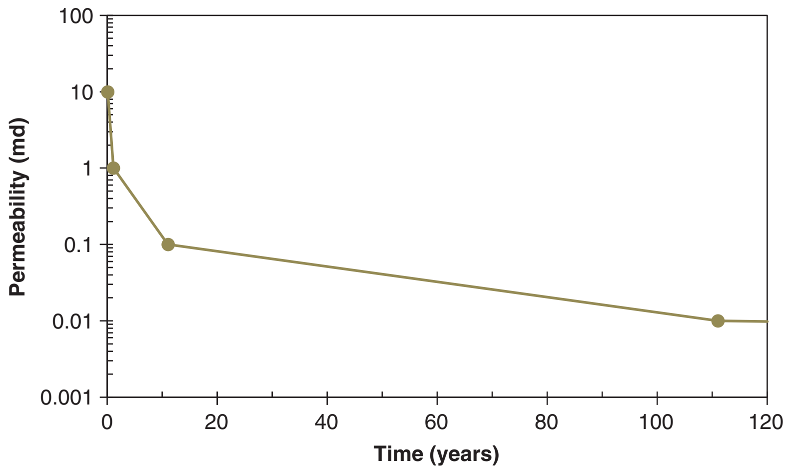


Fig. 1.2—Transient conditions can last for an extraordinarily long time for the very-low-permeability gas case.

for moderate-permeability and moderate-compressibility reservoirs. Also, the lower the compressibility, (the less gassy), the sooner boundary effects are encountered.

The more permeable layer in a noncommunicating, multizone completion becomes affected by the outer boundary within a shorter length of time than the less-permeable layers.

In conclusion, we can say that changing boundary conditions in variable-permeability reservoirs can cause the *b*-exponent to remain at high values and appear transient in nature, though it eventually trends toward zero.

Low-Pressure Gas Example. **Fig. 1.2** represents time in years required for a low-pressure (1,000 psia) dry gas reservoir to reach boundary-dominated-flow conditions as a function of average permeability. It is apparent that well life, although probably producing at a low rate for the very-low-permeability case, can last for years.

Assumptions. The following assumptions anticipate that a production history follows an unaltered and smooth decline. However, operational variations often divide a production history into segments, each reflecting different constant bottomhole pressure and production rate. These relations may be caused by

- The assumption that well flow is not mechanically restricted by chokes or tubing size. In actuality, production records often do not record choke changes. Dramatic flow rate changes signify something.
- Reservoir depletion conditions that remain relatively constant. Operational changes such as completing or abandoning wells might alter well drainage area which in turn can change performance characteristics. One should be careful about evaluating a production curve extending over a long period if history is not smoothly declining.
- Sufficient production performance data are available, and a declining trend has been established under boundary-dominated flow conditions; i.e., the well is draining a constant drainage area (pseudosteady flow if bottomhole pressure is constant).
- The well is produced at or near capacity. The productivity index of the well does not change. Factors that influenced the performance trends of wells or fields in the past will continue to govern their performance in the same manner.
- Absence of water influx or gas-cap expansion. Addition of an extra energy source must be considered when predicting future behavior.

Shapes of Production Decline Curves. Fig. 1.3 shows semilog rated time decline curves for four different wells located in the same field, but of different producing abilities. Line curvature defines future performance.

Fig. 1.4 shows how fitting a series of straight lines between two data points can provide a basis for predicting future production. The slopes of the declining straight lines constantly decrease.

Arps Equations

Arps (1945) modeled the various average shape of a line concepts to form a unified approach. The location of the fitted curve is defined in space by three values to form the equation of a line. These values are

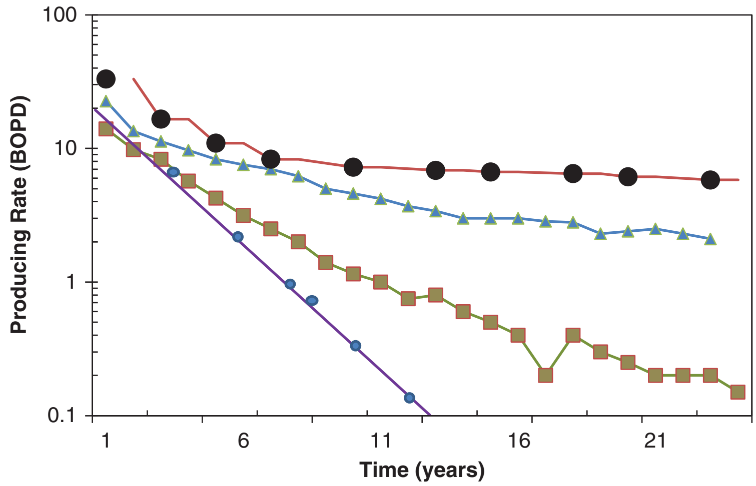


Fig. 1.3—Definitive shapes defined by the Arps equation. It can be observed that the top two curves never decline to a zero production rate, indicating transient flow conditions possible because of commingled layers (no crossflow) and Arps should be applied with caution. Decline curves are normally presented on a semilog rate vs. time plot.

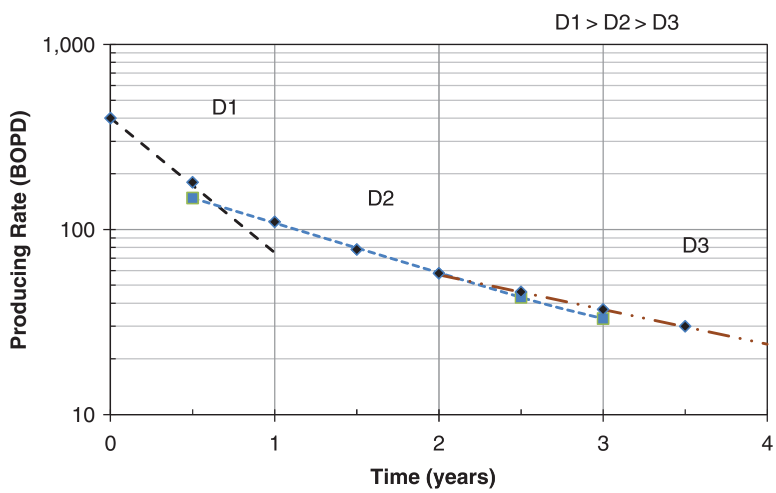


Fig. 1.4—Divide a curved semilog rate vs. time plot into a series of chords of decreasing slope. The declining straight-line slopes form a smoothly declining curve in the case for Fig. 1.2.

- Some initial producing rate (q_i)
- An initial decline rate (d_i) (might or might not coincide with the field data)
- The degree of curvature of the declining line (a function of the (b-exponent) term)

Arps (1945) defined the loss ratio as $-a = \dfrac{q}{\dfrac{dq}{dt}}$ or $a = -\dfrac{q}{\dfrac{dq}{dt}}$ . . . . (1.2)

The reciprocal of the loss ratio is defined as decline rate (D). where $a = 1/D$.

Therefore, $-\dfrac{1}{D} = \dfrac{q}{\dfrac{dq}{dt}}$ . . . . (1.3)

The loss ratio derivative, (the tangent slope of the line) is (b), where

$$b = \frac{d(1/D)}{dt} = \frac{d}{dt}\left(\frac{q}{dq/dt}\right) \quad (1.4)$$

Arps (1945) further defined that an exponential curve occurs when a series of rate/time estimates exhibit a constant (b) value and a hyperbolic curve when the derivative of the loss ratio remains constant.

Integrating over time results in a relationship between time, changing decline rates, and the b-exponent term:

$$D = \frac{D_i}{1 + bD_i t} \quad (1.5)$$

Substituting the Arps definition given in Eq. 1.3 into previous equation 1.5 results in

$$-\frac{d(\ln q)}{dt} = \frac{D_i}{1 + bD_i t} \quad (1.6)$$

Substituting the Arps definition and integrating from ($0 \rightarrow t$) develops an exponential rate vs. time expression

$$q_2 = q_1 \exp(-Dt) \quad (1.7)$$

Please note $b = 0$. Integrating over ($0 \rightarrow t$) develops a hyperbolic rate-time expression:

$$q_2 = \frac{q_1}{(1 + btD_i)^{1/b}} \quad (1.8)$$

Eq. 1.8 reduces to Eq. 1.9 for the harmonic case,

$$q_2 = \frac{q_1}{(1 + tD_i)} \quad (1.9)$$

Please note $b = 1$.

Exponential Decline. The following develops two relationships for the exponential decline.

Constant Percentage Exponential Decline. Apply a stepwise definition for an exponential decline. The effective or constant rate decline expresses incremental rate loss as a stepwise function. Define the first rate as (q_1) and a subsequent rate as (q_2). The rate differences usually span 1 year. Be wary of a decline rate expressed in a lesser time span.

$$d = \frac{q_1 - q_2}{q_1} = -\frac{\frac{\Delta q}{\Delta t}}{q_1} = 1/\text{time} \quad (1.10)$$

Rearrange to $t = \dfrac{\ln \frac{q_2}{q_1}}{-\ln(1-d)}$ (1.11)

Rearrange to develop a rate equation:

$$q_2 = q_1 (1-d)^t \quad (1.12)$$

Integrate from ($t_i \rightarrow t_2$) and obtain cumulative production:

$$Q_p = \frac{q_1 - q_2}{-\ln(1-d)} \quad (1.13)$$

Convention assumes the decline rate is expressed in terms of %/yr.

Decline rates expressed in monthly units might be a subterfuge to force a well exhibiting a dramatic production falloff to appear in a better light.

Including the b-exponent term presents a major problem when adjusting the time-unit span. Monthly and daily decline rate equations are:

Convert from rate/year to rate/month:

$$d_y = 1 - (1 - d_m)^{12} \quad (1.14a)$$

Convert from rate/year to rate/day:

$$d_y = 1 - (1 - d_d)^{365} \quad (1.14b)$$

Example. When 220 BOPD is 12 months and 63 BOPD is 24 months are interpreted from a performance curve, calculate constant percentage decline rate.

$$d = \frac{q_1 - q_2}{q_1} = \frac{(220 - 63)(100)}{220} = 71.4\%\,/\text{yr}$$

Convert the decline rate time units from %/yr to %/month:

$$(1 - 0.714) = (1 - d_m)^{12}, \text{or } d_m = (0.099)(100) = 9.9\%\,/\text{month}$$

Arps Nominal Decline. Arps nominal or continuous rate decline is considered here.

Arps (1945) and Brons* (personal communication) expressed the rate of change in the flow rate as a function of decline rate (D). Rearrange the exponential rate equation (Eq. 1.11) to solve for the decline rate:

$$D = \frac{\ln(q_1 / q_2)}{t} \quad (1.15)$$

Rearrange to provide a time interval relationship:

$$t = \frac{\ln(q_1 / q_2)}{D} \quad (1.16)$$

Substituting rate expression and integrating over integral limits 0 to t result in a cumulative production expression:

$$Q_p = \frac{q_1 - q_2}{D} \quad (1.17)$$

Use estimated ultimate recovery (EUR) to estimate the theoretical maximum reserves. $EUR = Q_{max}$; assume $q_{last} = 0$ in the limit as t goes to infinity, for exponential decline. Please note this is not the economic limit (EL).

$$Q_{\max} = EUR = \frac{q_1}{D} \quad (1.18)$$

Comparing Constant and Continuous Declines. A rewritten form of the effective decline definition is

$$\frac{q_2}{q_1} = 1 - d \quad (1.19)$$

Rewrite nominal decline definition as $\frac{q_2}{q_1} = \exp(-Dt)$. (1.20)

Combining results in

$$d = 1 - \exp(-Dt) \quad (1.21a)$$

Or, conversely:

$$D = -\ln(1 - d) \quad (1.21b)$$

This development shows that the exponential decline definitions are different but will produce similar answers if the proper equations are applied.

The solid line in **Fig. 1.5** compares relative decline rate values for the constant percentage and continuous decline definitions. A 45° slope existing up to a 25% decline reflects a similarity between the two different methods. However, the continuous decline rate increases quite dramatically when compared to the constant percentage decline rate values after this point.

In conclusion, we see the exponential curve may be defined in the context of an effective or a nominal decline. The equations are different, but the results of the calculations are the same. Either can be applied to study exponential decline curves if the proper equation sets are applied.

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*Brons, F. Personal Communication, 1966.

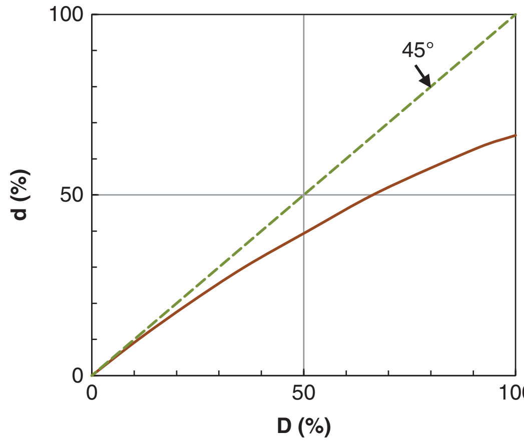


Fig. 1.5—Comparing effective and continuous decline rates. Note the similarity of the values up to approximately $D = 25\%$ value.

	Effective (Constant Percentage)	Continuous (Nominal)
Decline rate	$d = \frac{q_1 - q_2}{q_1}$	$D = \frac{\ln\left(\frac{q_1}{q_2}\right)}{t}$
Producing rate	$q_2 = q_1(1-d)^t$	$q_2 = q_1 \exp(-Dt)$
Elapsed time	$t = \frac{\ln\left(\frac{q_2}{q_1}\right)}{-\ln(1-d)}$	$t = \frac{\ln\left(\frac{q_1}{q_2}\right)}{D}$
Cumulative recovery	$Q_p = \frac{q_1 - q_2}{-\ln(1-d)}$	$Q_p = \frac{q_1 - q_2}{D}$
EUR	$Q_p = \frac{q_1}{-\ln(1-d)}$	$Q_p = \frac{q_1}{D}$

Table 1.2—Comparison of Effective and Continuous Decline Equations.

Rate vs. Time Plot. Express the exponential rate equation in logarithmic terms and arrange in the form of the equation of a straight line. See **Fig. 1.6.**

$$\ln q_2 = -Dt + \ln q_i \qquad (1.22)$$

Components

Plotting Variables	Outcome Variables
"y" axes: ($\ln q_2$)	"y" intercept: ($\ln q_i$)
"x" axis: (t)	the slope of the line is: ($-D$)

Table 1.3—Components of Rate vs. Time Plot Exponential decline.

Rate vs. Cumulative Production Plot. Rearrange the cumulative production equation to the equation of a straight line:

$$q_2 = -Q_p D + q_i \qquad (1.23)$$

Components

Plotting Variables	Outcome Variables
"y" axes: (q_2)	"y" intercept: (q_i)
"x" axis: (Q_p)	slope of the line is: ($-D$)

Table 1.4—Components of Rate vs. Cumulative Production Plot Exponential decline.

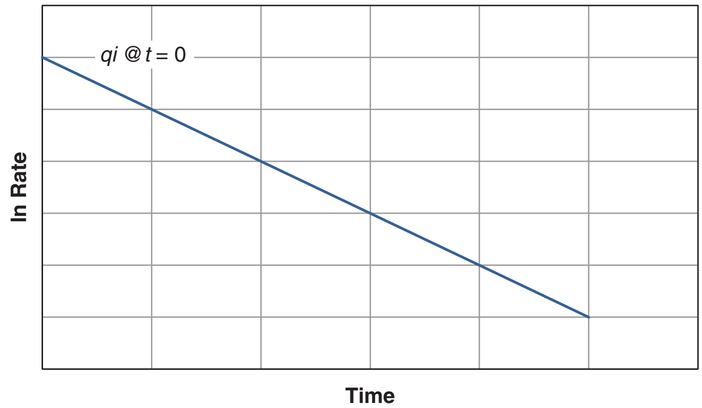


Fig. 1.6—The well-known exponential logarithmic rate vs. time plot which is the usual initial plot for all decline curve analysis. Predict future performance by extrapolating along the straight line. Note value of *qi* at *t* = 0.

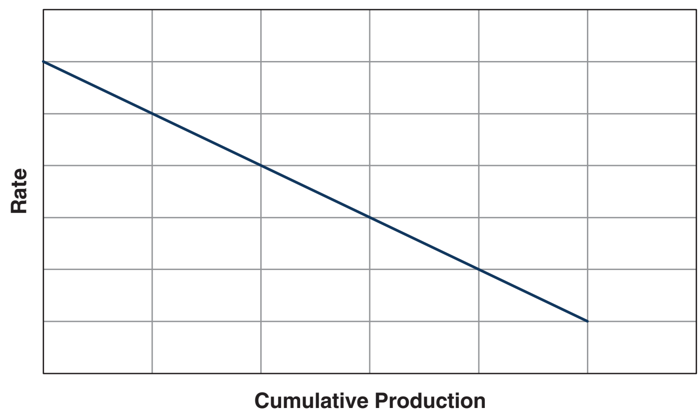


Fig. 1.7—A straight line plot for a rate vs. time curve represents an exponential decline.

Extrapolating a straight line through a (q vs. Q_p) plot results in an estimate of cumulative recovery for the exponential curve **(Fig. 1.7).**

In conclusion, we can say that straight line semilog rate vs. time and Cartesian rate vs. cumulative production plots define the presence of an exponential decline.

Combine the continuous decline rate and Arps (1945) (b-exponent) definition.

$$D = -\frac{1}{q}\frac{\mathrm{d}q}{\mathrm{d}t} = D_i\left(\frac{q}{q_i}\right)^b \quad (1.24)$$

Arps Hyperbolic Equations When 0 < b ≤ 1. Recall Arps (1945) defined the hyperbolic case to encompass the ($0 < b < 1$) range and reduced the general rate equation to

$$q_2 = \frac{q_i}{(1+btD_i)^{1/b}}. \quad (1.25)$$

Rearranging Eq. 1.24,

$$t = \frac{\left(\frac{q_i}{q_2}\right)^b - 1}{bD_i} \quad (1.26)$$

A rate-decline rate relationship is given by

$$\frac{D_i}{D_2} = \left(\frac{q_i}{q_2}\right)^b \quad (1.27)$$

Substitute the rate equation and integrate; Q_p is the integral of $q(t)$ with respect to t for $(0 < b < 1)$. This results in

$$Q_p = \frac{q_i}{D_i(1-b)}\left[1 - \frac{1}{(1-bD_i t)^{(1-b)/b}}\right] \quad (1.28a)$$

Substituting the rate equation simplifies to

$$Q_p = \frac{q_i^b}{D_i(1-b)}(q_i^{1-b} - q_2^{1-b}) \quad (1.28b)$$

Assume ($q_2 = 0$) to express in terms of a theoretical maximum recovery estimate (*EUR*) which is not the EL. Please note that b has to be < 1.

$$Q_{\max} = \frac{q_i^b}{D_i(1-b)}(q_i^{1-b}) \quad (1.29)$$

Harmonic Equations. The harmonic case is a restricted version of a hyperbolic case when the exponent term is defined as (b = 1).

The hyperbolic equation reverts to

$$\frac{D_i}{D_1} = \frac{q_i}{q_2} \quad (1.30)$$

The previously defined harmonic rate equation is

$$q_2 = \frac{q_i}{1 + D_i t} \quad (1.31)$$

Rearrange the harmonic rate equation to determine the time difference spanning two rates.

$$t = \frac{q_1 - q_2}{D_i q_2} \quad (1.32)$$

To combine rate and time and integrate, use

$$Q_p = \frac{q_i}{D_i}\ln(1 + D_i t) \quad (1.33a)$$

Combine with the Arps definition to simplify:

$$Q_p = \frac{q_i}{D_i}\ln\frac{q_i}{q_2} \quad (1.33b)$$

The Straight-Line Plot. Rewrite the rate equation to a straight-line relationship **(Fig. 1.8).**

$$q_2 = \ln q_2 - \frac{Q_p D_i}{q_i} \quad (1.34)$$

Straight Line Components

Plotting Variables	Outcome Variables
"*y*" axes: (ln *q*)	"*y*" intercept: ($\ln q_i$)
"*x*" axis: (Q_p).	slope of the line is: $\left(-\frac{D_i}{q_i}\right)$

Table 1.5—Components of Rate vs. Cumulative Production Plot.

For the Arps equations,

- An exponential ($b = 0$) line models single-phase flow from a pressure-depleting reservoir.
- Hyperbolic curves ($0 < b < 1$) model multilayered, gas, or multiphase-flow reservoirs.
- Harmonic ($b = 1$) curves indicate continued presence of transient conditions.

A low value b-exponent indicating eventual decline to a zero rate reflects when boundary flow affects predominate. On the other hand, harmonic and (b > 1) values indicate that transient conditions remain and a quantitative reserves estimate is problematical.

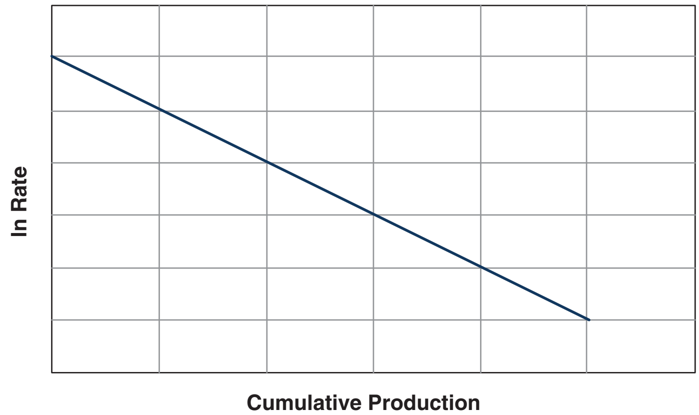


Fig. 1.8—A straight line results when the cumulative recovery equation is rearranged in the form of a straight line.

	Exponential	Hyperbolic	Harmonic
	$b = 0$	$0 < b < 1$	$b = 1$
D	$\frac{\ln(q_1/q_2)}{t}$	$D_i\left(\frac{q_2}{q_i}\right)^{1/b}$	$D_i\frac{q_2}{q_i}$
q	$q_1\exp(-Dt)$	$\frac{q_i}{(1+btD_i)^{1/b}}$	$\frac{q_i}{(1+tD_i)}$
Q_p	$\frac{q_1-q_2}{D}$	$Q_p=\frac{q_i^b}{D_i(1-b)}(q_i^{1-b}-q_2^{1-b})$	$\frac{q_i}{D_i}\ln(1+D_it)$
t	$\frac{\ln(q_1/q_2)}{D}$	$\frac{\left(\frac{q_1}{q_2}\right)^b-1}{bD_i}$	$\frac{q_1-q_2}{D_iq_2}$
EUR	$\frac{q_1}{D}$	$\frac{q_i}{D_i(1-b)}$; (No q @ EL, so restricted to 0<b<1).	$\frac{q_i}{D_i}\ln(1+D_it)$

Table 1.6—The Arps exponential, hyperbolic, and harmonic rate, time, cumulative production, and decline rate equations.

Bounds of the Arps Equations. Theoretically, the b-exponent term included in the Arps (1945) rate vs. time equation could vary in a positive or negative manner. However, a negative b-exponent value implies an increasing production rate. Therefore, the Arps (1945) equations are truly appropriate only within $(0 < b < 1)$ bounds.

Substituting $(b \geq 1)$ into the hyperbolic rate equation implies the decline rate is always increasing. This is a nonstarter.

$$q=\frac{q_i}{D_i(1-b)}\left(1-\frac{q_2}{q_i}\right)^{1-b} \quad (1.35)$$

In conclusion, we can say only exponential and hyperbolic declines converge to zero because the integral of $q(t)$ dt is a finite integral (as t goes to infinity for $b < 1$).

When comparing the general semilog rate vs. time plot for the Arps exponential, hyperbolic, harmonic, and $b > 1$ equations:

- The Arps exponential and hyperbolic rate vs. time curves trend in a downward manner to eventually attain a zero rate.
- The harmonic curve does not converge to zero but comes close.
- $b \geq 1$ values do not converge and confirm continuing transient flow generally from a highly variable permeability producing section.

Transient Boundary-Dominated Conditions

An indicator of the drainage radius expanding to boundary dominated flow is the log rate vs. log time plot shown in **Fig. 1.9.** The shape of the log-log plot is a function of the ultimate drainage volume and permeability distribution of the dual porosity system. The transient side usually produces a ½-slope signifying predominantly fracture flow, while the unit slope signifies that the drainage boundary has been reached.

Initializing Decline Curves. Production rates can change because of external and internal factors. When flow occurs after a well or field is temporarily shut in, rates are higher than normal because of the buildup of close-to-wellbore storage. Eventually, production reverts to boundary-dominated conditions after this unsteady state production is unloaded. These effects might impart a segmented curve whose long-term history mirrors the actual depletion history.

Fetkovich (1986) shows in **Fig. 1.10** the effects on the production rate when a North Sea field was periodically shut in. Pronounced production spikes are evident when the closed wells are opened back up. However, the rate soon returns to the normal field decline after inflow has returned to normal rates.

Glenn Pool Field Example. Production history of the Glenn Pool Field in Oklahoma is illustrated in **Table 1.7** and **Fig. 1.11** (Cutler 1924).

Divide the production history into the shaded columns shown in Table 1.7 to compare before and after depletion mechanisms.

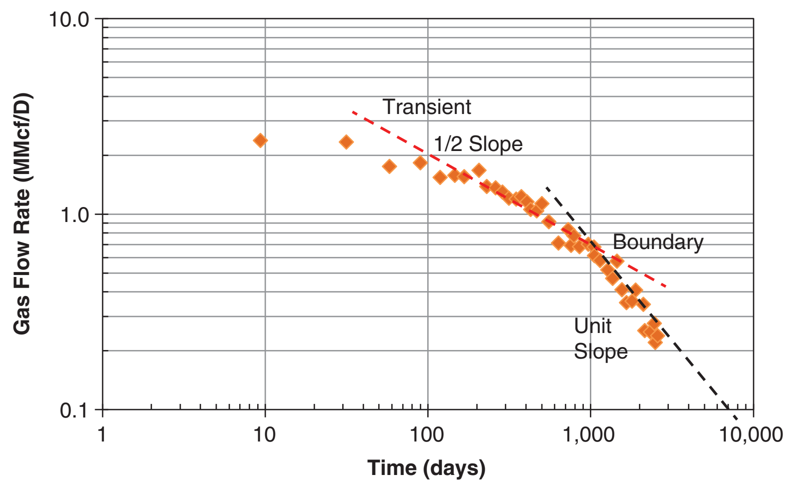


Fig. 1.9—The log rate- log time plot divides well performance history into transient and boundary-dominated depletion regimes. This plot is particularly important when studying low-permeability wells.

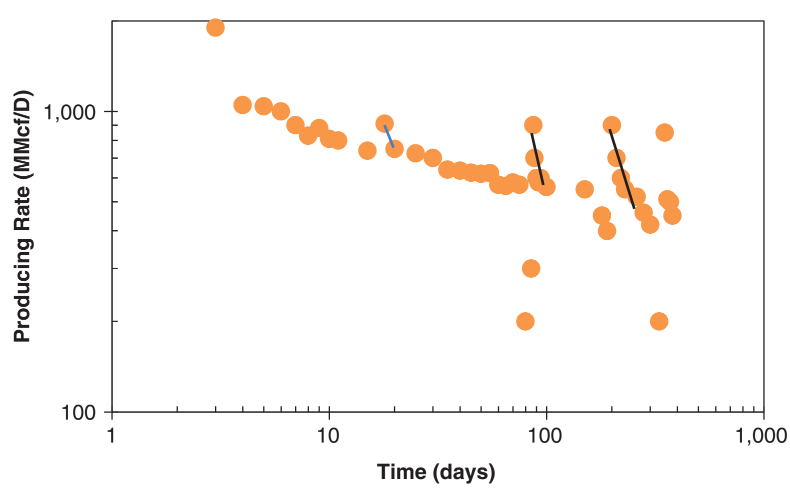


Fig. 1.10—Depletion history for the North Sea field. In each instance production soon declined to the field depletion rate after being reopened to flow. Adapted from Fetkovich (1980).

Time (year)	Initialized Rate	Rate (BOPY)
1	1	10000
2	2	6000
3	3	3400
4	4	2400
5	5	1500
6		1700
7		1850
8		1800
9	1	1750
10	2	1150
11	3	700
12	4	500
13	5	400
14	6	290
15	7	220

Table 1.7—Glenn Pool Field production data.

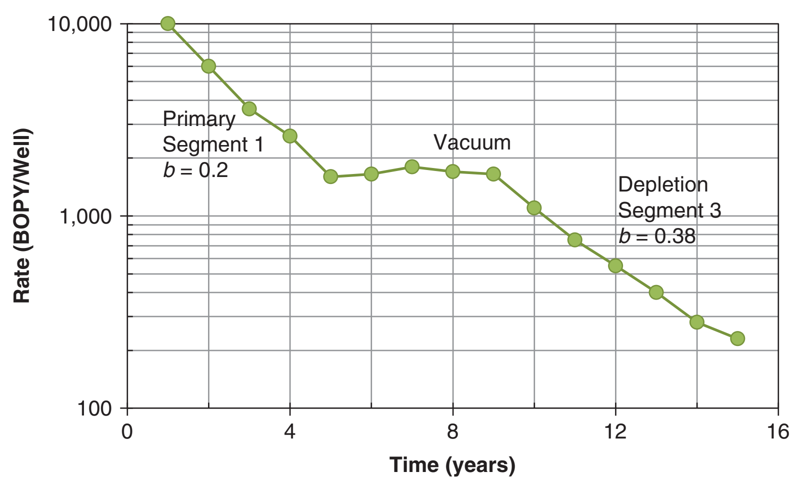


Fig. 1.11—Depletion history of the Glen Pool Field. Adapted from Cutler (1924).

- Segment 1 consists of the initial production decline curve that occurred as the field was depleted to essentially atmospheric pressure.
- Decline ceased at Year 5 when the production system was placed on vacuum and remained essentially constant to Year 9.
- Segment 3 commenced when vacuum operations were discontinued and normal recovery methods were reinstituted. What can we interpret from the production history?

Divide production history into "Primary—Segment 1" and "Depletion—Segment 3" to determine if reservoir depletion reverted to the original mechanism after vacuum operations ceased.

Highlighted values in Table 1.7 reflect two selected production periods. The primary data set was fit to the Arps curve while the Segment 3 set was initialized starting at (t = 1) and then fitted to an Arps curve. A good match was obtained for both cases.

Table 1.8 compares the results of analysis. Performance histories of the Glen Pool Field indicated that the vacuum operation produced additional oil. Comparing production history by reinitializing shows that the field has reverted to a hyperbolic decline and that instituting vacuum operations probably accelerated field depletion.

Segment	*b*	*Di*	*qi*
1	0.20	0.64	18,421
2	0.38	0.65	3,170

Table 1.8—Comparison of the decline characteristics for the two segments.

Reserves to Production Ratio. The reserves to production ratio, (R/P) provides a handy screening tool to predict performance when information is scarce.

$$\frac{Reserves}{Production} = \frac{Q_p}{q_{last}} \quad (1.36)$$

Related to the exponential (EUR) equation,

$$\frac{Q_p}{q_{last}} = \frac{1}{D} \quad (1.37)$$

Please note that this is EUR and not EL.

The value provides a useful screening tool to evaluate the well quality. Most of the wells should cluster in the middle of the plot, but good and bad wells that should require further evaluation are located at the ends of the spectrum.

Reserves are calculated by decline curve analysis or by some other means.

PROBLEM(S)

Example Problem 1.1. Apply exponential decline curve analysis techniques to analyze the Wafford No. 1 well rate vs. time history. See **Fig. P.1.1.1.**

Learning Objectives. Realize applying either the nominal or the effective exponential decline equations results in similar answers, and understand fitting a line to production data is often an individual selection process. Please answer the following questions.

1. Draw a best fit straight-line approximation of the performance history.
2. Determine effective decline rate. Compare decline rate over a 1-year period.
3. Determine the nominal decline rate.
4. Compare the two answers.
5. Calculate expected producing rate at Month 28?
6. How much longer will the well produce when the economic limit is 10 BOPM?
7. How much oil will be produced between Month 28 and the 10-BOPM economic limit?

Example Problem 1.2. Apply the exponential concept to calculate the effect of well clean out on performance.

Learning Objective. Apply exponential rate and cumulative production equations to evaluate reserves potential for working over the Hollands No. 3A well.

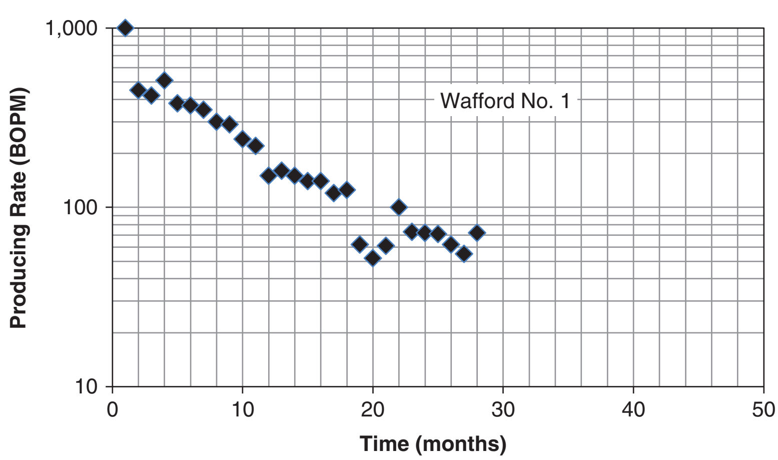


Fig. P1.1.1—Producing history of Wafford No. 1 well.

The Hollands No. 3A well currently displays a D = 37%/yr decline rate and produces at 52 BOPD. Replacing the pump and scraping the producing string would increase the rate from 52 to 96 BOPD but not change the reserves picture. This is a rate acceleration problem. Economic Limit = 6 BOPD.

Compare a “Do Nothing” case to the “Remedial” case to calculate the economics of the projected workover expense.

Useful equations: $N_p = \dfrac{q_1 - q_2}{D}$, $\quad D = \dfrac{\ln(q_1 / q_2)}{t}$.

Hints:

- The endpoint for the “Do Nothing” and “Remedial” cases is the volume of oil that could theoretically be produced to the estimated ultimate recovery (EUR).
- Calculate (EUR) by assuming $q_{\text{last}} = 0$.

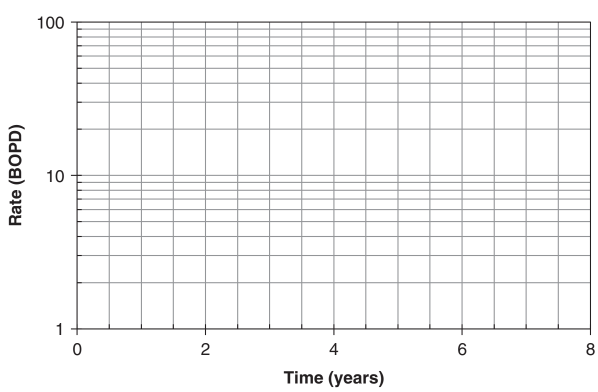
Set cumulative production for the “Do Nothing” and “Remedial” cases equal to each other. Calculate the decline rate for the “Remedial Case”, as shown in Table 1.9.

- Apply the system of equal triangles to determine the new decline rate.
- Calculate a new rate vs. time forecast to compare with the old forecast.

Year	Do Nothing		Remedial		Incremental
	Rate (BOPM)	Cum. (BO)	Rate (BOPM)	Cum. (BO)	(BO)
0	52		96		
1	36	15,784	49	25,117	9,333

Table 1.9—Comparison table.

Calculate and plot the rate vs. time schedule on the following graph.



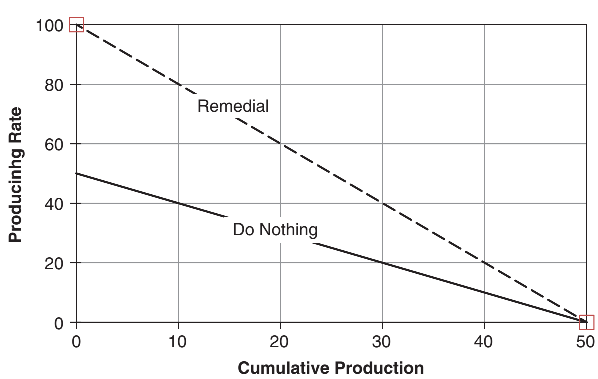


Fig. P1.2.1—Producing history of Well #3A.

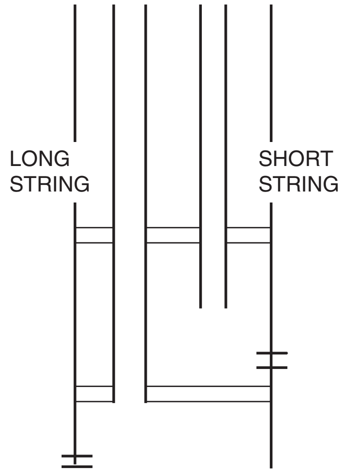


Fig. P1.3.1—Completion setup of a dually completed gas well.

Example Problem 1.3. Diagnosing a well problem.

Learning Objective. A study of production records can aid in interpretation of source of excess water production.

A gas well was dually completed in two pay zones at approximately 5,500 ft: generally, sands in this area are friable. In fact, both completions do produce some sand. **Fig. P1.3.1** shows the completion setup with the lower (long string) and upper (short string) set of perforations separated by a packer.

Recently water production has been observed in the long string, and the rate has declined to approach that of the upper sand.

Is this effect caused by normal water encroachment in the reservoir or by a hole eroded into the blast joint?

Interpret performance histories of a dually completed well to find the unwanted water source as well as obtaining insight into reservoir performance.

Fig. P1.3.2 compares gas production rates for the two zones. Note the erratic gas production rate from the short string.

Fig. P1.3.3 shows the track of the flowing-tubinghead pressure (FTHP) for the two completion zones. Water production was consistently measured.

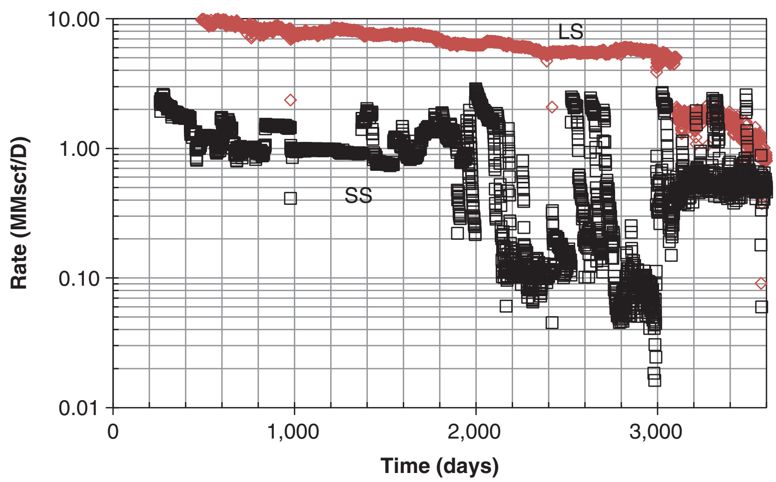


Fig. P1.3.2—Notice the long string (LS) produced at a much higher rate than the short string (SS). Decline rate for the deeper well, *D* = 5%/yr. The short string experienced sanding problems over much of its producing life. Well problems caused erratic production rates after approximately 1,900 days.

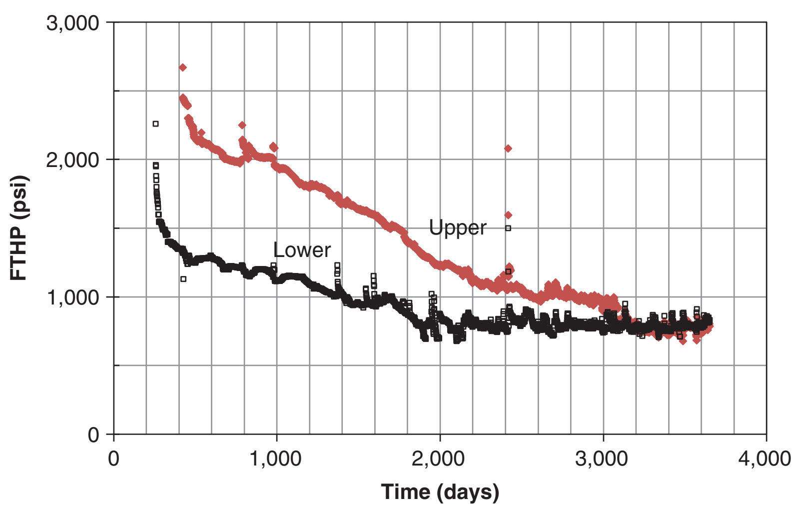


Fig. P1.3.3—Flowing-tubinghead-pressure history for the two completions.

Consider these questions:

- What is your interpretation of the histories of the two completions?
- Does water encroachment affect reservoir performance?
- Can you estimate when communication between the two production strings began?

Example Problem 1.4. Has the well watered out or is there a hole in the tubing?

Learning Objective. Couple well locations with performance analysis to determine source of unforeseen water production.

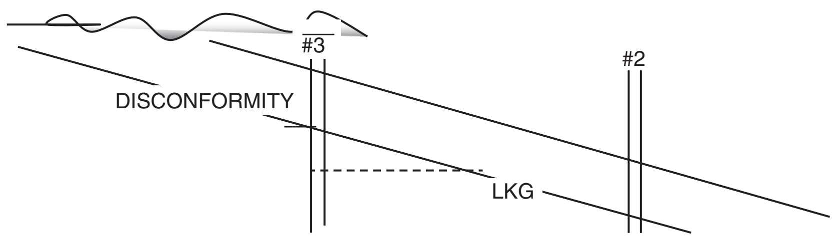


Fig. P1.4.1—Well locations and structural interpretation.

Fig. P1.4.1 shows that the #2 well was drilled in a downdip location (defined by an unconformity) in a layered, friable sand. A lowest known gas (LKG) was observed. Perforations were at the top of the sand. However the #2 well recently watered out.

One year later, the updip #3 well was drilled and encountered a gas-filled sand similar in nature to that of #2. The well was completed and inflow performance 34 MMscf/D with no water.

History shown in the **Fig. P1.4.2** indicates that the #2 well produced at 20 MMscf/D until 1,000 days when it sanded up and was down for approximately 600 days. The well never returned to its initial potential after gravel pack.

On day 1,800, the #2 well began to cut water and 2.2 years later was shut in because of high water production.

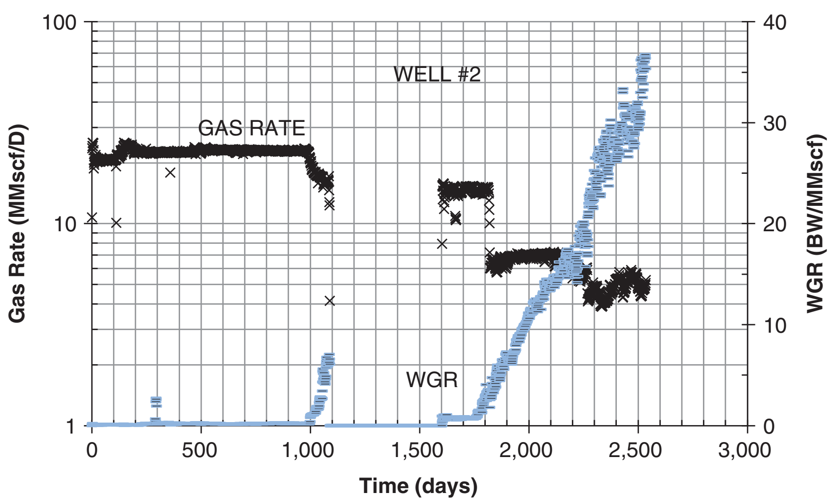


Fig. P1.4.2—Producing history of Well #2.

The updip #3 well continues to produce at approximately 20 MMscf/D essentially water free. **(Fig. P1.4.3)**.

Example Problem 1.5. A north texas gas condensate well.

Fig. P1.5.1 shows the 5-year history for the gas-condensate well.

Learning Objectives. Relate changes in field operations to changes in the shape of the decline curve. Please answer the following questions.

1. Two workovers occurred during the life of the well. Can you spot the probable time of these workovers? Were they effective?

2. What can you say about the consistency of the gas and condensate producing rates?
3. What eventually killed the well?
4. Was there a hole in the tubing?
5. What is the probable cause for the increased water production early in the well life?

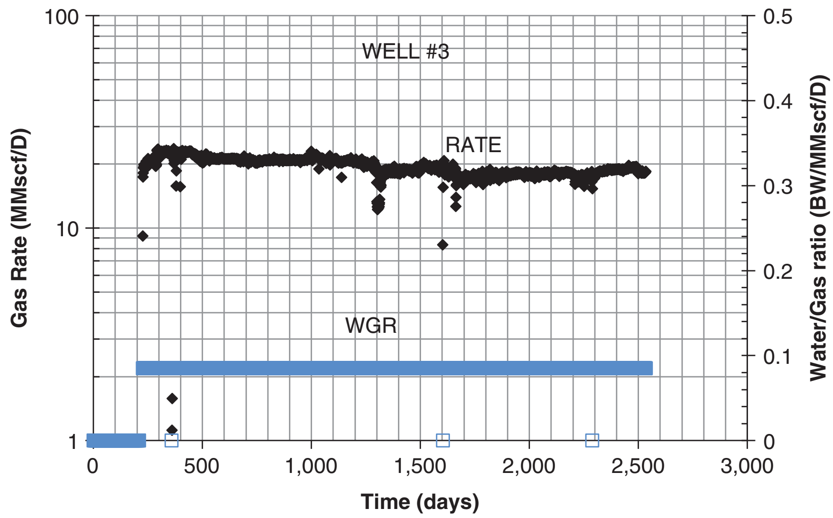


Fig. P1.4.3—History of well #3.

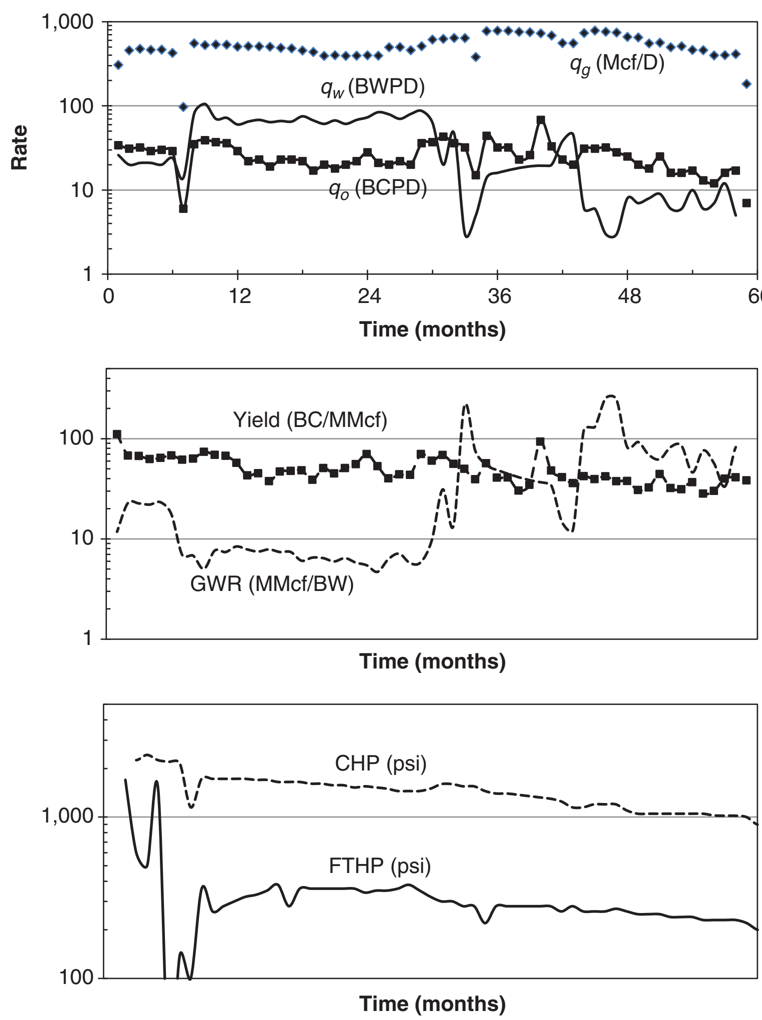


Fig. P1.5.1—A north Texas gas/condensate well. CHP = Casinghead pressure.