Essentials of Hydraulic Fracturing
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Preface

In 1989, the publication SPE Monograph V. 12, Recent Advances in Hydraulic Fracturing, addressed four decades of fracturing technology. Since then, hydraulic fracturing has moved from vertical wellbore, massive hydraulic fracturing in tight microdarcy gas reservoirs to new frontiers addressing horizontal wellbore fracturing in massive nanodarcy formations. Accordingly, the industry has kept pace with advances in extended horizontal drilling and fracturing applications. New fracturing materials, techniques, and applications have emerged. However, the fundamental basics of fracture propagation behavior and diagnostics remain the same.

This book focuses on consolidating the old and the new in a format that assists both current and future fracturing design engineers in their practice. It is beyond the scope of this book to extensively cover the entire gamut of fracturing intricacies; rather, the purpose is to provide [1] a basic understanding of (a) fracture propagation behavior, (b) the effects of fracturing on post-treatment well production, and (c) the important aspects pertinent to fracture treatment design application and [2] insight and methods for applying that knowledge to achieve maximum economic returns from a fracturing treatment.

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1

Introduction

This book focuses more on the “how to” than the theoretical aspects of fracture treatment design and execution. Much of what is contained here is also presented more esoterically and comprehensively in the SPE monograph cited here:


Additionally, fracturing advances that have emerged since monograph publication are presented herein.

The SPE monograph, though published in 1989, covers basic theoretical fundamentals and approaches, and basic fracturing materials available at that time. The basic fundamentals seldom change, nor do the properties and behaviors of materials found in nature. The monograph exhaustively and esoterically addresses them. While it’s not necessary for readers to have a copy of the monograph, it is a good resource for reference.

Many other books with basic information are also available and worth having in one’s technical library, such as


In the SPE monograph, and to some degree in the other books referenced above, the discussion of similar fracturing aspects are located in more than one chapter and in the appendices. Hence, efforts have been made in this book to consolidate, as much as possible, discussions pertinent to each separate aspect of fracture design into a single chapter.

Hopefully, the authors’ consolidation efforts, and those made to put the presentation into a practical application format, will facilitate design engineers in their efforts to apply state-of-the-art practices for fracturing treatment designs.
The contents of this book are intended to serve
- Design engineers currently involved in fracturing applications
- As a textbook for university engineering students
- Engineers designated for future fracturing involvement
- Line managers responsible for economic returns from fracturing

What it presents includes
- Aspects that are basic to treatment design
- Their effects (singularly and interactively) on fracture propagation and performance behavior
- Their relative impact on post-frac production and revenue
- Algorithms and examples pertinent to treatment design and analysis
- Fracturing treatment design methods and processes
- Pre- and post-fracturing approaches and diagnostics for evaluating treatment performance, and for improving performance on future wells

The intended purpose of the book is to serve the reader in several ways, including but not limited to providing
- An understanding of how basic factors and phenomena pertinent to fracture propagation geometry and fracture conductivity impact the results of a treatment
- Awareness of important considerations pertinent to treatment design and execution
- A menu of data requirements and procedures necessary to design and analyze treatments
- Methods and procedures for processing design data and creating designs
- Encouragement to communicate with all entities associated with fracturing the target well, including: management, geologists, geophysicists, reservoir engineers, computer modelers, consultants, field operating personnel, service companies, equipment and material suppliers, etc.
- A focus on the most important goals of hydraulic fracturing, i.e.,
  - Safety
  - Environmental prudence
  - Maximum economic returns

Hydraulic Fracturing

The following discussion is obviously oversimplified for experienced design engineers. However, it is cast as such to provide insight to those less familiar with the topic of hydraulic fracturing. Many fracturing treatments are unique to specific wells and to the design addressing them. It benefits even the experienced to revisit those mentioned in the simplified discussion.

Also, after some engineers have been involved in hydraulic fracturing for several years, especially in a given locale, they may develop somewhat of a “fracturing expert” posture. This may,
Refracturing of Previously Hydraulically Fractured Wells

Refracturing of previously fractured wells has been common practice throughout the history of hydraulic fracturing. Refracturing pertains to wells that were successfully stimulated, and then produced until the oil and gas flow rates declined to an uncomfortably low level, as opposed to redoing an unsuccessful treatment. Economic and/or logistic success has ranged by varying degrees from poor to excellent. This has been for a variety of reasons. Although refracturing per se is not covered in this book, several aspects pertinent to refracturing are addressed. The primary one being in situ stress reduction in the target fracturing interval by virtue of reservoir pressure depletion. With a refracturing treatment, the reduced in situ stress from reduced pore pressure results in wider fractures and more vertical growth confinement. There is another stress-related issue that comes into play: the in situ stress field imposed by the residual proppant pack width. This can affect refracture propagation behavior. Other issues are pertinent, such as the inhibiting effects of in-place proppant packs on slurry flow.

Environmental Impacts of Hydraulic Fracturing Treatments

Environmental issues have entered the fracturing world to a significant degree. They will continue to be a large factor in the environmentally friendly development of wells that need to be fracture-treated. To do it right requires a familiarity with pertinent regulations at all governmental levels and that corporate communication lines pertinent to environmental aspects are open and active.

Recently, public interest has grown and may possibly continue doing so about several issues, such as

- Does hydraulic fracturing affect drinking water aquifers?
- Does hydraulic fracturing cause issues with earthquakes?

These two issues are being debated as the authors write this book, so there will be much information to come in the following years. Design engineers have some control over the drinking water issue. These are discussed. In regard to earthquakes, there is not sufficient definitive information on a global basis to serve as guidance for a design engineer. Consequently, the issue is pending credible investigations pertinent to specific locales. Disposal is emerging as the cause.

Prospective subsurface fresh water issues?

The following addresses two more commonly discussed fresh water pollution questions that are repeatedly discussed as possible causes of fresh water aquifer issues.

- Wellbore annulus invasion—prospective vertically upward fluid migration outside the casing?
- Prospective fracture vertical growth?
Note: In figure 2–2, propped vertical fracture height (hf) equals net pay height (hi), so the entire net pay is effectively propped. Tinsley et al. presents other charts where net pay is not totally propped. Consequences for this are obvious: lower FOI. This emphasizes the importance of treatment designs that always effectively prop the entire pay.

![Fig. 2–2. Tinsley et al. FOI chart](image)

Source: “Figure 1.4,” Gidley et al. 1989, 2.

In the chart, it is easily seen that as ki decreases, RC increases (to the right), and higher values of the productivity index ratio or FOIs can be achieved with deeper fracture penetrations (Xf/Re). For RC > 100, FOI is governed primarily by fracture penetration. For RC > 500, FOI remains essentially constant, regardless of RC magnitude.

However, as ki increases, RC decreases (to the left), and here, FC dominates. Note that for RC < 1, fracture penetration has little impact on FOI. In this range, maximum achievable values are FOI = 2 or lower.

As a general rule of thumb for a semi-steady state reservoir flow, low-permeability formations require deeply penetrating fractures to increase the productivity index. For these low-permeability reservoirs, fracture conductivity is not as important as fracture length as long as sufficient conductivity exists for the fracture fluid to clean up in the fracture. Conversely, the optimum fracture for high permeability formations generally consists of shorter but higher conductivity fractures to provide sufficient permeability contrast with the formation. Hence, an accurate estimate of formation permeability prior to designing the fracture treatment is essential for success.

Production estimates for transient reservoir flow. The charts in figure 2–1 apply only to semi-steady state reservoir flow. If a reservoir exhibits transient flow over extended periods, the rule of thumb mentioned above, may not be completely applicable, but the principal of requiring deep penetrating fractures in low-permeability reservoirs remains valid. The problem is that the
Basic equations and associated figures pertinent to the above parameters are introduced in this chapter to provide a general perspective of their effects on fracture propagation behavior. Some are also included in chapter 3, “Rock Mechanics and Fracture Propagation,” along with expanded discussion, additional equations, figures, tables, and example calculations.

**Data sources—values for treatment design**

Figure 2–11 provides a perception of how the various formation properties interact in fracture propagation behavior. The following discussion explains how the design engineer can determine the values for the various rock properties and parameters required to compute fracture dimensions.

**Overburden pressure**

The value of the overburden stress is best obtained using downhole wellbore logs for interval thickness and formation density, since the value of the overburden stress at any depth is simply the weight of all the rocks and fluids above that point. Overburden at the top of any given interval can be calculated using equation 2–13.

\[
P_{\text{OVRB}} = \Sigma_{\text{I}}(\text{N})\left[\rho_{\text{GRAD}}(\text{I})h_{\text{GROSS}}(\text{I})\right] \text{ Equation 2–13}
\]

where

- \(P_{\text{OVRB}}\) Overburden pressure at the interval top
- \(N\) Number of overlying intervals
- \(\rho_{\text{GRAD}}(\text{I})\) Average density depth gradient of the \(I\)th interval
- \(h_{\text{GROSS}}(\text{I})\) Thickness of the \(I\)th interval

\(P_{\text{OVRB}}\) is often calculated as the product of an assumed average \(\rho_{\text{GRAD}}\) (typically ranging from 0.9 to 1.1 psi/ft) multiplied by the total depth to the formation top. This estimate then serves as a starting point for in situ stress calculations in the underlying target fracturing interval.

**Elastic modulus (E) and Poisson’s ratio (\(\nu\))**

These values are obtained from either:

- Downhole compression and shear acoustic full-waveform logs (dynamic measurements);
- Laboratory measurements on cores (static measurements); or
- Published sources, obtained for a specified lithology.

Dynamic measurements (full-waveform) are made using sonic signals at relatively high frequencies. Dynamic measurements can be made using sonic logs or using sonic measurements on cores in the laboratory. Static measurements are made using stress-strain tests on cores in the laboratory.

Full-waveform acoustic logs provide by far the most usable data. They span all the layers of rock that are logged and evaluated. The results from the logs reflect the rock behavior under in situ confinement conditions. Additionally, the results represent a much larger formation sampling
e.g., "It's a power-law fluid." This is attributed to available test results spanning specified, common in-fracture shear rate ranges where the fluid system exhibits, more or less, a single behavior. This is possibly to the chagrin of classical rheologists. In spite of the fact that a given fluid system exhibits multiple rheology behaviors under different shear rate ranges, the vernacular prevails, and is generally accepted by the industry.

Figure 2–20 depicts an example that shows the two following aspects:

- Several rheology category behaviors for a given system
- Shear rate and temperature effects on apparent viscosity

The chart data covers a much wider shear rate spectrum than is conventionally available per standard test procedures. The fluid system comprises an aqueous-based 40 lbm/1,000 gal (40 parts/M-gal) hydroxypropyl guar (HPG) polymer fracturing fluid with no cross-linking agent.

**Fig. 2–20. Fluid viscosity versus shear rate**
*Source: “Figure 9.8,” Gidley et al. 1989, 186.*

Figure 2–20 shows a range of different viscosity behaviors over shear rates of $10^{-1} < \dot{\gamma} < 10^0$. Within that range, there are four behaviors:

- **Shear rates**: $\dot{\gamma} < 10^{-1}$ 1/sec: Newtonian—No, or minimal, effect of shear rate on apparent viscosity. Apparent viscosity $\approx$ constant at a given temperature
- **Shear rates**: $10^{-1} < \dot{\gamma} < 10^{1}$ 1/sec: Non-power law—Logarithmically nonlinear. Apparent viscosity does not behave per equation 2–19
- **Shear rates**: $10^{1} < \dot{\gamma} < 10^{2}$ 1/sec: Near-power law—Logarithmically nearly linear. Apparent viscosity $\approx$ equation 2–19
- **Shear rates**: $10^{2} < \dot{\gamma}$ 1/sec: Power law—Logarithmically linear. Apparent viscosity behaves per equation 2–19

The industry has become more aware that apparent viscosity flattens to a constant viscosity as shear rate decreases. However, this behavior may or may not be incorporated in some fracture
Essentials of Hydraulic Fracturing

- Lithology
- Rock mechanical properties
  - Poisson’s ratio
  - Elastic modus
  - Fracture toughness
- In situ stress
- Net fracturing pressures
- Interval interface slippage effects

The above list is by no means exhaustive. Discussion in this chapter addresses individual aspects, along with the interactive effects of one parameter upon the others, and the effects imposed by fluid injection forces. Design engineers should strive to

- Acquire the most credible information that is economically possible
- Develop a thorough understanding of the basic concepts and the interdependencies pertinent to the interaction of all relevant parameters

### Mechanical Rock Properties and In Situ Stresses in Fracture Propagation Models

How rock mechanics are integrated into fracture design models may seem somewhat obscure. The complexities in some models may seem somewhat convoluted. To simplify this, two long-standing, less complicated models are used for discussions. These have been used successfully for design since the advent of fracturing models and are depicted in figure 3–1.

- Perkins–Kern–Nordgren (PKN), which applies where total fracture length is greater than or equal to total fracture height
- Geertma and deKlerk (GdK), which applies where total fracture length is less than or equal to total fracture height

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**Fig. 3–1.** Perkins–Kern–Nordgren (PKN) and Geertma–deKlerk (GdK) models. Copyright 1989, SPE. Reproduced with permission of SPE. Further reproduction prohibited without permission.