

Second Edition

Project Economics and Decision Analysis

Volume 2: Probabilistic Models

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PennWell®

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Preface

Welcome to the second edition of *Project Economics & Decision Analysis, Volume 2: Probabilistic Models*. As mentioned in the previous edition, the international oil and gas industry remains one of the most important, highly capital-intensive and risky industries at global, regional, and local levels. The global exploration and production (E&P) expenditure reached \$324 billion in 2007 (*World Oil Magazine*, February 2008, Vol. 229, No. 2). Our profit margins are under real pressure from many factors, including the higher costs of developing new reserves, less oil found per foot drilled, rising inflationary costs of doing business, oversupply of crude, crude oil price volatility, competition for oil company investments, competition for acreage/concessions, competition for funds, and the overall business risk and uncertainty.

Therefore, it is crucial to carry out prudent economic evaluations of any capital investments before resources are committed. This, of course, requires a thorough understanding of the techniques available and their application by all those involved in decision making. To assist in achieving this goal, the industry deserves a comprehensive guide to provide all the necessary concepts of capital investment evaluation, capital budgeting, and decision analysis. This edition of volume 2 along with *Project Economics and Decision Analysis, Volume 1: Deterministic Models* will hopefully meet this requirement.

Objective

One goal in writing this book has been to provide students, practicing engineers, geologists, economists, planners, and managers with a solid foundation in the dynamic and growing field of capital investment evaluation, with emphasis on the uncertainty aspect. It describes how investment decisions are currently made under different stages of uncertainty and prescribes techniques for making rational decisions.

This two-volume set describes the philosophy, process, and methods of capital investment evaluation and decision analysis. In summary, the main objectives of the book are to:

- Explain the ever-expanding role of economics in prudent capital investment decision making.
- Assist readers in developing a knowledgeable vocabulary of the terms associated with economic analysis.
- Review the procedures used in preparing capital investment evaluations and decision analysis.

- Relate the new vocabulary and knowledge to some specific problems.
- Present ways of interpreting estimates that include uncertainty (e.g., converting probabilistic description into a measure of profitability).
- Provide solid hands-on experience with capital investment evaluation and decision analysis tools and processes.

The subject of project economics and decision analysis is not difficult. It does not require you to remember differential equations or understand nuclear physics or atomic theory. However, it does require the use of a good deal of logic and common sense; the rest is just number crunching. The analysts making economic evaluations need to think imaginatively, identify a problem and its variables, weigh the various factors that will affect these variables, evolve alternatives, take a position, and finally be prepared to defend that position.

I urge students and practitioners to take a helicopter view of the entire process and develop an eye for details while resolving problems as opposed to completing a series of cookbook computations. This is what *Project Economics and Decision Analysis* is all about. While I have retained most of the features that have made these books so popular, I have further enhanced the coverage to make them more relevant and accessible to students of varying backgrounds.

This edition is further improved in order to aid readers who are unfamiliar with the subject. An attempt has been made to present the subject matter clearly and simply so that the reader will be in a position, whether in the classroom or in practice, to move immediately to higher levels of sophistication in understanding and application.

Changes in the Second Edition

This edition continues the basic philosophy of the first edition, which is to provide users with coverage of all important areas of project economics and decision analysis, while providing flexibility in the use of materials. This objective of flexibility has warranted the inclusion of some new materials.

Those who will be using these books as college textbooks, learning guides, or as a reference in day-to-day economic decision making may be interested to know what is different about the second edition. Apart from polishing the material and making the concepts easier to understand, the following changes are of interest:

1. All materials that have a time aspect have been updated.
2. Throughout the book I have added new, real-world examples to round out the coverage of concepts and to provide appropriate emphasis on areas of central importance.
3. Although many readers will be proficient in the use of Microsoft Excel, others will be less aware of the power and limitations of this powerful software application, especially when it comes to its tools that are of particular help in economic evaluation of projects. Therefore, I have expanded the spreadsheet application sections to elaborate on the capabilities of Excel and highlight the areas most valuable for analysts.
4. The discussion about the weighted average cost of capital (WACC) concept is expanded to better reveal the weakness of using this concept in practice.
5. A new section on netback pricing and indexed netback pricing is added.
6. The explanation of unit technical cost, also referred to as long-run marginal cost (LRMC), is expanded to make its value easier to calculate and apply.
7. The section on international economics (fiscal systems) is rewritten and expanded.
8. A new chapter on exploration economics is added to volume 2.
9. Problems in volume 2 that were solved using the previous versions of the Palisade DecisionTools™ suite are updated using the latest version of the suite.
10. After using the books for six years, with significant feedback from the participants in my five-day courses on the subject and some of the professors who are using these books in their courses, I found that there was room for improvement in adding clarity to some of the topics. Therefore, I have provided additional examples or reworked the existing examples.
11. For professors who are using these books, a test bank of approximately 300 multiple-choice and true-false questions has been developed. In addition, a comprehensive set of more than 300 PowerPoint™ slides is now available.

Emphasis and Style

This book presents a balanced blend of theoretical concepts and their practical utility. In my experience, extensive theoretical discussions can distract the reader from the most important concepts and their practical applications. Moreover, theory can seem sterile and pointless unless its usefulness is made clear. Therefore, in this volume I focus more on practical application. The underlying concepts are stressed and made concrete with liberal use of illustrations, many of them taken from real-life capital investment evaluations. Algebraic formulations and spreadsheets are shown side-by-side to help readers develop conceptual thinking skills. Emphasis is placed on model formulation and interpretation rather than algorithms.

The technical materials have been developed with considerable patience—assuming very little foreknowledge and proceeding in a step-by-step manner to minimize the possibility of the reader getting lost along the way. Moreover, a degree of informality is used in the presentations in the belief that *readability* should be an overriding consideration. Toward the same goal, intuitive and economic explanations are frequently provided to clarify the *why* of a particular concept or technique.

This book is primarily intended for use by economists, earth scientists, engineers, and students. It also is intended to serve as a refresher or a self-study textbook. The problem-solving approach is instructive in nature, but the foundational principles show the practical application of the material. Its chief purpose is two-fold: (1) to render a systematic exposition of certain basic deterministic investment evaluation methods, and (2) to relate these to the decision analysis in such a way that the mutual relevance of the two is clearly brought out.

Therefore, *Project Economics and Decision Analysis* is divided into two separate, yet complementary, volumes. Volume 1 is essentially introductory and deals with the *deterministic* evaluation tools used for capital investment evaluations. These concepts are seldom covered as broadly or from the same viewpoints in economics and other courses, yet they are fundamental to the proper understanding of all evaluation work. Volume 2 deals with the concepts of decision analysis (i.e., incorporating risk and uncertainty as applied to capital investments). Generally, each topic is introduced by a practical or conceptual overview, followed by a brief discussion related to its application in practice and a solved example. The companion CD in this volume includes applications, spreadsheets, and tables that expand the practical application of the material in the book.

For optimum benefit, it is recommended that readers explore both volumes and benefit from their integrated instruction.

Examples and Assignment Problems

As in the first edition, this edition uses an example-driven approach to the subject matter. I believe that the best way to learn any subject is by working through examples and completing plenty of realistic, applicable problems. Although this active learning approach is not new, I believe these books have more fully developed this approach than any others on the subject. This example-driven approach is further reinforced by imparting valuable modeling skills that students can appreciate and take with them into their careers.

Included in this second edition are more than 100 additional solved real-life examples that help reinforce the learning process and over 300 end-of-chapter questions and problems that can be used for assignment material. Each solved example is straightforward, fully explained, and avoids sudden leaps to conclusions that might confuse the reader. The end-of-chapter questions primarily address key concepts and terms in the chapter. The problems either consolidate a number of chapter topics or focus on a comprehensive analysis of a single topic.

The wide variety of assignment material offers practical knowledge since the assignments include various combinations of breadth and depth, theory and procedures, simplicity and complexity. For maximum benefit, the reader should work out as many of these problems as possible, if indeed not all.

Spreadsheet Applications

As an instructor, I find that students often struggle with the use of the financial functions provided in Microsoft Excel. Today, most of us are using spreadsheet programs to build models of the decision problems we face as a regular part of our daily work activities. Spreadsheets also capture users' interest and add a new relevance to investment evaluation and decision analysis. Since we use Excel extensively for our economic modeling, it is essential that those who are interested in working in this area be fully conversant with Excel.

In this second edition, I have attempted to highlight the common mistakes committed even by senior analysts in modeling problems using MS Excel; such mistakes are unnecessary and avoidable. Rather than routinely using an existing spreadsheet calculation to solve a problem, I like to stress the importance of using software to organize and expedite our calculations. We should use a tool only if we know, given sufficient time, how to solve a given problem by hand. To this end, I have tried to show both methods of developing the required calculations: problem solving by hand and by using Excel. In order to build economic models you also need to know how to debug these

models, and in order to find bugs in any spreadsheet or economic model you need to understand the underlying mathematical concepts.

Examples are provided to show how spreadsheet applications can be used to help make better evaluations and, hence, better decisions. The tools provided in Microsoft Excel make it increasingly easy and practical to do sensitivity and scenario analysis. Other Excel add-in tools such as DecisionTools™ Suite by Palisade Corp., including the award winning @RISK and PrecisionTree, are used and explained where applicable.

Reviewers' Comments

Many useful comments were received from PennWell's technical reviewers, and I have incorporated their suggestions wherever possible. Some reviewers noted that perhaps I may overkill some of the basic concepts, such as the time value of money. That may be true, but this is how I learned these concepts more than 30 years ago when I took a course from Dr. Franklin J. Stermole at the Colorado School of Mines. It is ironic that it is these simple concepts that very experienced analysts make mistakes or misinterpret the limitation of the concepts applied. I have specifically brought these mistakes to your attention so that they can be avoided. These basic concepts have been of considerable help to me over the years, enabling me to visualize the investment problems and their time horizons. Therefore, I have decided to leave them as is. Similarly, the use of interest tables (in this modern age of computers) might be considered ultra orthodox. The tables may not be used in practice, but they definitely add to understanding the concepts. I feel strongly that readers will benefit from their continued inclusion.

Request for Suggestions

It should be noted that one cannot always add new material without the need to delete some of the existing material. Some examples and materials become dated and become natural candidates for deletion. In addition, some material needs to be enhanced by rewriting or restructuring it in order to improve readability and understanding.

A considerable amount of dedication and investment (time and capital) goes into writing and publishing such a book. I have made every effort to introduce this two-volume set as a comprehensive desk reference. I sincerely welcome your thoughts as an end user to help us further improve the content, presentation, and utility of this book so as to make it a standard for the new generation of petroleum industry personnel. I will always be very grateful for your comments, suggestions, or corrections sent to me directly (asifmian55@yahoo.com) or through PennWell.

1

Introduction

In volume 1 of this series, it was assumed that the results and implications of investment decisions are known at the time the decisions are made, meaning the future is known and it is known with certainty. In reality, such conditions simply do not exist. In practice, different investment options have inherently different degrees of associated uncertainty and consequently different degrees of risk. The terms *uncertainty* and *risk* are formally defined in the following pages. However, the deterministic concepts presented in volume 1 (as will be shown in this chapter) form an integral part of decision analysis, such as:

- Profit planning and some basic principles of economics
- The time value of money concepts
- Before-tax and after-tax cash-flow models
- International petroleum economics (concessionary systems and production-sharing agreements)
- Capital budgeting techniques
- Investment selection decision making

Uncertainty about the variables in the investment model complicates the decision-making process. Many important decisions have to be made without knowing exactly what will happen in the future or exactly what the ultimate outcome will be from a decision made today. This introductory chapter sets a stage, by presenting basic concepts and definitions, for decision analysis techniques presented in the following chapters.

The purpose of decision analysis is to help a decision maker think systematically about complex problems and to improve the quality of the resulting decisions. In this regard, it is important to distinguish between a good decision and a lucky outcome. A good decision is one that is made based on a thorough understanding of the problem and careful thought regarding the important issues. Outcomes, on the other hand, may be unlucky, regardless of decision quality. Therefore, decision analysis allows effective decisions to be made more consistently.¹

Example 2–1

The performance of a certain type of rock bit in a given formation must be analyzed. After searching the drilling records for the pertinent data, the information shown in table 2–1 is gathered. Using this data, calculate the mean, median, and mode of the footage drilled by the bit under evaluation.¹

Table 2–1. Bit record for example 2–1

Bit No.	Footage Drilled, ft	Footage Drilled, ft
1	69	53
2	72	69
3	123	72
4	135	76
5	95	80
6	89	89
7	139	90
8	109	95
9	125	102
10	80	102
11	102	105
12	53	108
13	76	109
14	102	110
15	90	115
16	116	116
17	105	123
18	108	125
19	115	135
20	110	139
n = 20	$\Sigma X_i = 2,013$	

Solution: Column 2 of table 2–1 lists the footage drilled by each bit. This data is sorted in an ascending order of footage drilled in column 3.

Using equation (2.3), the mean is

$$\bar{X} = \frac{\sum_{i=1}^n X_i}{n} = \frac{2,013}{20} = 100.65 \text{ feet}$$

Coefficient of peakedness and skewness. These additional equations may be used to determine the peakedness and skewness of the frequency distributions. The coefficient of peakedness a_4 is dimensionless and computed as follows:

$$a_4 = \frac{\text{fourth central moment}}{s^4} = \frac{m_4}{s^4} \quad (2.28)$$

$$= \frac{\sum_{i=1}^n f_i (x_i - \bar{X})^4}{n s^4}$$

A dimensionless coefficient of skewness a_3 may be computed as

$$a_3 = \frac{\text{fourth central moment}}{s^3} = \frac{m_3}{s^3} \quad (2.29)$$

$$= \frac{\sum_{i=1}^n f_i (x_i - \bar{X})^3}{n s^3}$$

If

$a_3 = 0$, symmetric, the curve is bell shaped

$a_3 > 0$, the curve is skewed to the right

$a_3 < 0$, the curve is skewed to the left

Spreadsheet application

Most of the information presented in the preceding sections can be easily generated by the built-in capabilities of a spreadsheet application such as Microsoft Excel.

Table 2–8 shows the descriptive statistics of example 2–1 and example 2–2 for the data in table 2–1 as an Excel spreadsheet. When using a spreadsheet application like Excel, the data does not have to be arranged in ascending or descending order.

1. Construct the tree with branches representing all the possible events that can occur and write the *prior probabilities* for these events on the branches.
2. Attach a new branch to each branch representing the new information obtained or to be obtained. For example, there is 90% chance acquisition of seismic data will indicate structure when structure actually exists, and there is 10% chance it will indicate structure when structure actually does not exist. This additional information leads to additional branches of the probability tree.
3. Obtain the joint probabilities by multiplying each prior probability by the conditional probability that follows it on the tree.
4. Sum the joint probabilities.
5. Divide each joint probability by the sum of the joint probabilities to obtain the required *posterior probability*.

Using a table format. The table format is often more convenient than using the probability tree or the formula because the table format allows the information to be systematically displayed and manipulated. Secondly, any spreadsheet program can be used to apply Bayes’ rule using the table format. Table 2–12 and equation (2.38) relate directly to each other. The following steps summarize the Bayesian application via the table format.

Table 2–12. A Bayesian revision calculation procedure

Event	Prior Probability	Conditional Probability	Joint Probability	Posterior Probability
1	2	3	4 = 2 x 3	5 = 4/Σ(4)
A	$P(A)$	$P(X/A)$	$P(A) \times P(X/A)$	$P(A) \times P(X/A)/\Sigma(4)$
B	$P(B)$	$P(Y/B)$	$P(B) \times P(Y/B)$	$P(B) \times P(Y/B)/\Sigma(4)$
:	:	:	:	:
:	:	:	:	:
N	$P(N)$	$P(Z/N)$	$P(N) \times P(Z/N)$	$P(N) \times P(Z/N)/\Sigma(4)$
	$\Sigma(2) = 1.0$		$\Sigma(4)$	$\Sigma(5) = 1.0$

1. Input all the possible events in column 1 of table 2–12.
2. Assess the prior probability associated with each event and input these in column 2 (corresponding to the respective events). The sum of all probabilities in column 2 must equal 1.0.

Table 2–20. Using Excel's normal distribution and STANDARDIZE functions

	A	B	C
2	Upper Porosity Limit, %	15	
3	Lower Porosity Limit, %	12	
4	Average Porosity, %	12	
5	Standard Deviation of Porosity, %	2.5	
6			
7	Z at Upper Porosity Limit	1.20	=STANDARDIZE(B2,B\$4,B\$5)
8	Z at Lower Porosity Limit	0.00	=STANDARDIZE(B3,B\$4,B\$5)
9			
10	Probability of Porosity		
11	Between Mean and Upper Limit	38.49%	=NORMSDIST(B7)-0.5
12	Greater than Upper Limit	11.51%	=0.5-B11
13	Less than Upper Limit	88.49%	=0.5+B11
14	Between Upper and Lower Limits	38.49%	=NORMSDIST(B7)-NORMSDIST(B8)
15	Between Mean and Lower Limit	0.00%	=0.5-NORMSDIST(B8)
16	Greater than Lower Limit	50.00%	=NORMSDIST(B8)
17	Less than Lower Limit	50.00%	=0.5-B15

- a. Input the upper limit of porosity in cell B2, the lower limit of porosity in cell B3, the average porosity in cell B4, and the standard deviation of porosity in cell B5.
- b. Enter the formula =STANDARDIZE(B2,B\$4,B\$5) in cell B7 and copy it to cell B8. The standard normal deviate (z) is calculated for the upper and lower limits of porosities.
- c. Enter the function =NORMSDIST(B7)-0.50 in cell B11 to calculate the probability of porosity between the lower and upper limits. For the probability of porosity greater than the upper limit, enter =0.50-B11 in cell B12. Similarly, enter =0.50+B11 in cell B13, enter =NORMSDIST(B7) -NORMSDIST(B8) in cell B14, enter =0.50-NORMSDIST(B8) in cell B15, enter =NORMSDIST(B8) in cell B16, and enter =0.50-B15 in cell B17.

LOGNORMDIST. This function returns the cumulative lognormal distribution of random variable X, where $\ln(X)$ is normally distributed with the parameters mean and standard deviation. The syntax of the function is LOGNORMDIST(X, mean, standard deviation).

Example 3–2

A drilling prospect is evaluated with an estimate that the probability of a successful well is 35% and the probability of a dry hole is 65%. Drilling a dry hole will result in a net loss of \$250,000. If the well is successful, then the net present value (NPV) of the future streams of net cash flow (NCF) will be \$500,000. Instead of drilling the well, the prospect can be farmed out (i.e., no exposure to any costs) and retain an overriding royalty interest in the well. The NPV of farmout is estimated at \$50,000, the amount received in the form of royalty. Is it economically better to drill the well or farm it out?

Solution: The problem is solved in the payoff table of table 3–3.

Table 3–3. Payoff table for the data in example 3–2

	A	B	C	D	E	F
2	Outcome State	Probability	Drill		Farmout	
3			NPV	EMV	NPV	EMV
4	Dry Hole	0.65	(250,000)	(162,500)	0	0
5	Producer	0.35	500,000	175,000	50,000	17,500
6		1.00		12,500		17,500
7	Cell D4	=B4*C4	Copied to Cell D5			
8	Cell F4	=E4*B4	Copied to Cell F5			
9	Cell B6	=SUM(B4:B5)	Copied to Cell D6 and Cell F6			
10	Alternatively, Using the SUMPRODUCT Function					
11	EMV _{Drill}	12,500	=SUMPRODUCT(B4:B5,C4:C5)			
12	EMV _{Farmout}	17,500	=SUMPRODUCT(B4:B5,E4:E5)			

In example 3–2, the EMV = \$17,500 of farmout is higher than the EMV = \$12,500 of drilling. In order to maximize the expected net present value, it is prudent to farm out in this situation. The farmout is giving a net advantage of \$5,000 in expected present value profit over the drill option. However, would the decision hold if the probability of a producer were increased to 36% versus the 35%? Increasing the probability of the producer to 36% switches the decision against the farmout option. This shows that if one is not certain about the assigned probabilities, performing sensitivity analysis will give a range of probabilities in which each option will be feasible. Furthermore, the variance of the drill option is significantly higher than the variance of the farmout option (the drilling option is much more risky).