Reservoir Simulation: Problems and Solutions

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Dedication

This book is dedicated to current and past petroleum-engineering students and practitioners of the world who have made petroleum engineering their career choice.

If I had an hour to solve a problem and my life depended on the solution, I would spend the first 55 minutes determining the proper question to ask. For once I know the proper question, I could solve the problem in less than 5 minutes.

—*Albert Einstein*

Preface

In solving a reservoir simulation problem, the very first step will involve showing a good understanding of the problem description. Upon successful problem identification, some prioritization of strategies toward the solution of the problem will be necessary. Once these two steps are correctly executed, the final step will involve the implementation of a formal solution protocol. To set the stage for solving a reservoir simulation problem, it will be necessary to ensure that a set of criteria is adopted to check the plausibility of the problem (well-posed problem). In many areas of reservoir simulation, as it happens in many areas of mathematics, a problem for the application of a certain numerical protocol is not considered to be well-formulated unless the existence of a unique solution is assured and the solution is stable. Once the solution is generated, it will be necessary to analyze and reflect on the solution.

As in any other engineering and scientific area, solving reservoir simulation problems is important because each time we solve a problem, we learn how to make decisions independently, which in turn generates self-confidence and builds self-esteem, both of which feed into developing creativity, persistence, and a proactive mind set to prepare ourselves for problems of the real world. But, reservoir simulation problem solving goes deeper than this because it involves powerful imagination, requiring the reservoir simulation problem solver to take all of what one has learned and incorporate it into seemingly unrelated actions. It is widely accepted that simulation problem solving has grounds in fact-learning, but merely knowing facts does not make one a good reservoir simulation practitioner. In other words, a good reservoir simulation engineer or scientist is one who assembles and puts to use all the tools at his/her disposal. As a result, solutions to reservoir simulation problems should come as a complete package, with components such as basic skills, concept understanding, literacy in mathematics, and problem-solving philosophy. Then, it can be said that solving a reservoir simulation problem is knowing what to do when you do not immediately know what to do.

Reservoir simulation has been in practice for more than 50 years and has been gaining significant momentum with its wider applications in increasingly more-complex reservoir systems. Accordingly, we believe that the timing of this new book is well in agreement with the developing industry practices and academic curricula. It is the authors' view that, most of the time, reservoir simulation technology has been treated in a prescriptive manner. We are hopeful that this new book will bring reservoir simulation methodology and its intricacies much closer to the readers, allowing them to grasp the ideas more effectively. In this book, we provide solutions to the exercises that were presented in the *Basic Applied Reservoir Simulation* textbook authored by Turgay Ertekin, Jamal H. Abou-Kassem, and Gregory R. King (SPE Textbook Series, Volume 7, 2001). Accordingly, the overall outline of the book follows the original outline of the textbook. While this book contains solutions to approximately 180 exercises from the original book, it also introduces a new set of 180 exercises and their solutions, all of which come from the homework and examination sets used in our courses taught at Penn State University.

We do not know of the existence of a similar "problems and solutions" book in the petroleum-engineering literature, and therefore believe that this new book will be instrumental in structuring effective solution strategies to a large spectrum of reservoir simulation problems and will fill an existing gap. Petroleum-engineering undergraduate and graduate students and more-recent petroleum-engineering graduates will benefit from this book, and it is our hope that it will help students, young engineers, and earth scientists become astute problem solvers of reservoir simulation applications.

As you start walking the way, the way appears. —Rumi, 13th Century mystic poet

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May 2019

Acknowledgments

This book is a compendium of homework sets and midterm and final examinations of one senior-level and two graduate-level courses that we taught at Penn State University over a period of almost four decades. We owe a tremendous amount of gratitude to our students who have expended intense efforts to combine their creativity and understanding of the thematic topics of reservoir simulation to become better petroleum engineers. We also would like to extend our appreciation to many individuals from different parts of the world who have continuously encouraged us to write this book. We are indebted to the fine work and support of SPE Editorial Services Manager Ms. Jane Eden and SPE Editor Ms. Shashana Pearson-Hormilosa for guiding us, and perhaps more importantly, walking with us through the tortuous paths of producing a technical book of this kind. Finally, we thank the SPE Textbook Committee for inviting us to write this book and several colleagues of whom we do not know their names who have served as technical reviewers and provided much valuable feedback.

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Chapter 1

Introduction

Reservoir simulation is normally associated with modeling of bulk-fluid transport in porous media (macroscopic view), which is the subject of this book. With the recent inclusion of ultratight, nanoporous, and nanopermeability reservoirs in our resource portfolio, however, pore-scale molecular models, stochastic models, and models based on artificial intelligence are also of interest to the industry. One of the first questions that comes into a discussion about analytical methods and numerical methods is how do they compare when implemented into reservoir engineering applications? Perhaps a statement such as, "a numerical method provides an approximate solution to an exact problem, whereas an analytical method provides an exact solution to an approximate problem," describes the difference between the two formalisms in the most-effective way. However, one should realize that there is always some degree of exaggeration in the statement when the phrase "exact problem" is used. Mother Nature typically provides challenging and, to a certain extent, ill-posed problems (caused by our misunderstanding of certain processes). This is a result of the heterogeneous and anisotropic distribution of rock properties and nonlinear behavior of the fluid properties. Accordingly, in an analytical formulation it is necessary to remove some of these complexities, and to do so, we must incorporate some sweeping assumptions. As we do, we begin to deviate from the exact representation of a problem and end up with an approximate representation, which is a simplified version of the actual problem. Consider, for example, the classical well test model:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial p}{\partial r}\right)=\frac{\phi\mu c}{k}\frac{\partial p}{\partial t}, \quad \text{(1.1a)}$$

with typical boundary conditions stating constant flow rate and infinite-acting reservoir,

$$r\frac{\partial p}{\partial r}\bigg|_{r=r_w}=-\frac{q\mu}{2\pi kh}\bigg|_{r=r_w} \text{ for } t>0\text{, and as } r\to\infty,\ p\to p_i\text{, for } t\geq 0, \quad \text{(1.1b)}$$

and with the initial condition stating uniform pressure distribution over the reservoir domain to be tested,

$$p=p_i \text{ at } t=0,\ \text{for } r_w\leq r<\infty. \quad \text{(1.1c)}$$

Eqs. 1.1a through 1.1c represent an analytical model of fluid flow in porous media. A close inspection of these equations reveals the following assumptions are already in place:

- Flow is taking place in a 1D domain in radial-cylindrical coordinates.
- The flow domain is infinitely large, and there is only one well that is produced at a constant rate.
- When the well is put on production at a constant rate of q, this rate is accomplished instantaneously.
- The reservoir has a uniform pressure distribution at $t = 0$ (before the well is put on production).
- The reservoir has homogeneous and isotropic property distribution.
- Fluid properties are assumed to be constant and a single-phase flow condition is assumed to be in place.
- Gravitational forces are ignored.
- Flow regime is laminar, and flowing fluid exhibits Newtonian behavior.
- Isothermal flow conditions exist.

It is obvious that with the inclusion of the assumptions listed, the problem is rather simplified so an analytical solution to this approximate problem can be developed. This solution is then going to be an exact representation of pressure distribution within the approximate representation of the flow domain and flow conditions.

The numerical formulation honors most of the challenges and complexities imposed by Mother Nature. One can incorporate all the heterogeneities of reservoir characteristics together with the anisotropic flow parameters and simultaneously incorporate not only gravitational forces but also capillary forces. In ultratight reservoirs with slow fluid velocities, however, subgrid-scale heterogeneities may be of interest, but cannot be captured in numerical models. One can, however, incorporate multimechanistic flow concepts, such as Darcian and Fickian flow components can be simultaneously taken into consideration (Ertekin et al. 1986). Of course, pressure and temperature dependency of all the fluids can also be incorporated wherever they exist. There is no doubt that such a representation will enable engineers and geoscientists to come much closer to the exact representation in problem descriptions. However, with all these complexities in place, an analytical-solution protocol will not be feasible any more, forcing us toward a numerical-solution technique that will produce a solution inherently approximate in nature.

1.1 Analysis of Reservoir Dynamics and Plausible Methodologies

In a typical reservoir engineering application, the reservoir engineer or scientist always seeks to find a solution to a problem, which can be represented by Eq. 1.2:

$$q_{\psi}(t) = M(\Phi, \psi), \quad \text{(1.2)}$$

where $q_{\psi}(t)$ is the production and/or pressure characteristics as a function of time (time series); M (Φ, ψ) is a mathematical representation (mathematical model) of a fluid flow problem; Φ is the flow domain characteristics; and ψ is the flow process, including all the thermodynamics of the reservoir system.

In solving Eq. 1.2, the basic approaches that can be used are

- Deterministic modeling: The right-hand side of the equation, M, Φ, and ψ, is fully prescribed.
- Stochastic modeling: The right-hand side of the equation, M, Φ, and ψ, is partially known.
- Neurosimulation (machine learning): The right-hand side of the equation, M, Φ, and ψ, is not known, but the left-hand side is available in the form of a time-series data set (production history and/or pressure history). A neurosimulation approach can be implemented in forward calculations, as well as in two different forms of inverse calculations.

In all of these applications, the goal is to generate tractable, robust, and low-cost solutions. In achieving such solutions, the engineer expends efforts to exploit imprecision and uncertainty.

1.1.1 Systems Analysis. Using an equation such as Eq. 1.2 can be considered an implementation of a systems-analysis technique. A systems-analysis problem can be schematically represented, as shown in **Fig. 1.1**.

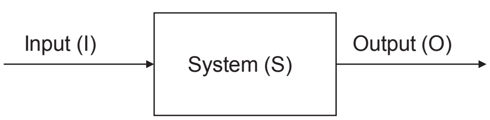

Fig. 1.1—Systems analysis.

In Fig. 1.1, the system, S, represents the reservoir (with its pertinent characteristics) and wellbore and the related parameters it is in communication with. The input, I, represents the boundary conditions (external and internal), which can be considered a forcing function applied to the system S. The response of the system to the imposed forcing function is the output, O, which is a time-series representation of the pressure surface or production surface.

The Forward Problem (Forecasting). If the systems analysis is implemented in the following form,

$$[I] \times [S] \rightarrow [O], \quad \text{(1.3)}$$

the solution of Eq. 1.3 will yield the response of a reservoir system to the imposed boundary and initial conditions. This analysis typically is considered a prediction or forecasting.

The Inverse Problem I (History Matching/Characterization). In an inverse problem, the engineer makes an attempt to find S from the known I and O, which are imposed and measured entities, respectively. Then, the systems analysis is carried out as solving the equation

$$[O] / [I] \rightarrow [S]. \quad \text{(1.4)}$$

The Inverse Problem II (Design, Strategy Development). Consider the implementation of an improved-oil-recovery project in a reservoir with known characteristics. The system properties are well known and the expected response, which is considered a feasible response, is also determined through a feasibility analysis (economic analysis). Then, the problem reduces to the strategic design and implementation of the project so that it will yield the desired outcome. In other words,

$$[O] / [S] \rightarrow [I]. \quad \text{(1.5)}$$

In Eq. 1.4, [I] contains the required design- and project-implementation-related parameters. It should be recognized that the solution of inverse equations (Eqs. 1.4 and 1.5) can generate nonunique results, so ultimately it is the project engineer's responsibility to find the unique solution on which he/she is focusing.

It is also important to realize that in reservoir modeling, a phrase such as, "for every complex problem, there is always a simple solution," does not always hold up, because that simple solution may be the wrong solution. Accordingly, in formulating a modeling approach, the schematic representation shown in **Fig. 1.2** can be helpful in formulating the problem.

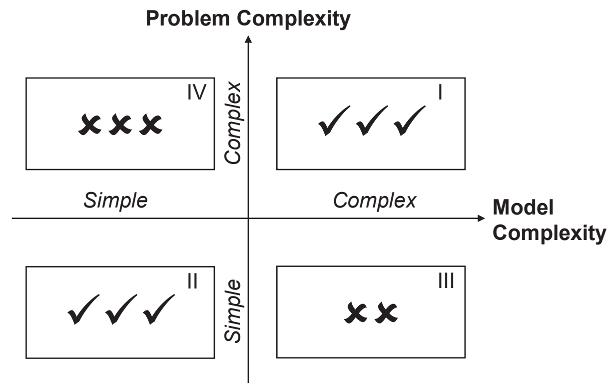

Fig. 1.2—Model complexity vs. problem complexity.

In Fig. 1.2, Quadrant I reminds us that as the problem complexity increases, the model sophistication accordingly should be increased. Quadrant III simply emphasizes the more-universal nature of the sophisticated models and their potential applications for simple problems. For example, a complex model can be used to generate solutions that can also be obtained by simple material-balance equations. Quadrant II simply states that for simple problems, one does not need sophisticated models. Quadrant IV underlines the impossibility of solving a complex problem with a simple model. Again, we should keep in mind the phrase, "for every complex problem, there is always a simple solution that is always wrong."

1.2 Problems and Solutions

Problem 1.2.1 (Ertekin et al. 2001)

What are the different ways a reservoir can be modeled?

Solution to Problem 1.2.1

- Traditional modeling approaches:
  - Analogical methods.
  - Experimental methods.
    - Analog models.
    - Physical models.
  - Mathematical methods.
    - Material-balance equations.
    - Decline-curve analysis.
    - Statistical approach.
    - Analytical approach.
- Reservoir simulation approach.

Problem 1.2.2 (Ertekin et al. 2001)

What are the differences between a mathematical, a numerical, and a computer model?

Solution to Problem 1.2.2

A numerical model is a class of mathematical models, and a computer model is an implementation of the numerical model for execution in a computational platform.

Problem 1.2.3 (Ertekin et al. 2001)

Match the items in the left column of **Table 1.1** with the corresponding item in the right-hand column.

Physical model	Simulator
Conceptual model	Partial-differential equations
Geological model	Material-balance equations
Mathematical model	Laboratory sandpacks
Computer model	Potentiometric model
Analog model	Depositional model
Numerical model	Empirical-correlation equations
Statistical model	Finite-difference equations

Table 1.1—Match the model on the left with the corresponding item on the right.

Solution to Problem 1.2.3

Table 1.2 displays the correct matches between models and their corresponding items.

Physical model	Laboratory sandpacks
Conceptual model	Partial-differential equations
Geological model	Depositional model
Mathematical model	Material-balance equations
Computer model	Simulator
Analog model	Potentiometric model
Numerical model	Finite-difference equations
Statistical model	Empirical-correlation equations

Table 1.2—Solution to Table 1.1: Models matched with their corresponding item.

Problem 1.2.4 (Ertekin et al. 2001)

To summarize the basic steps of a simulation study, put the following in sequential order:

- Define the study objectives.
- Prepare the data.
- Construct the geological model.
- History match.
- Predict performance.
- Analyze the results.
- Report.

Solution to Problem 1.2.4

1. Define the study objectives.
2. Construct the geological model.
3. Prepare the data.
4. Predict performance.
5. History match.
6. Analyze the results.
7. Report.

Problem 1.2.5 (Ertekin et al. 2001)

The equation $q = \dfrac{2\pi\beta_c kh\Delta p}{\mu \ln\left(\dfrac{r_e}{r_w}\right)}$ represents the steady-state radial flow of a fluid in a cylindrical porous medium. What are the analogous terms describing the heat and current flows in similar cylindrical systems? Identify the analogous terms and/or groups for

- Current flow: Ohm's law, $I = \frac{1}{R}\Delta E$.
- Heat Flow: $Q = \frac{KA\Delta T}{\Delta L}$.

Solution to Problem 1.2.5

Eqs. 1.6 through 1.8 express the analogous terms of Ohm's law:

$$I \sim q, \tag{1.6}$$

$$\frac{1}{R} \sim \frac{2\pi\beta_c hk}{\mu \ln\left(\frac{r_e}{r_w}\right)}, \tag{1.7}$$

$$\Delta E \sim \Delta p. \tag{1.8}$$

Eqs. 1.9 through 1.11 express the analogous terms of the heat flow equation:

$$Q \sim q, \tag{1.9}$$

$$\frac{KA}{\Delta L} \sim \frac{2\pi\beta_c hk}{\mu \ln\left(\frac{r_e}{r_w}\right)}, \tag{1.10}$$

$$\Delta T \sim \Delta p. \tag{1.11}$$

Problem in (Ertekin et al. 2001)

Comment on the accuracy of this statement: *The material-balance equation is considered a zero-dimensional model because time dependency is not incorporated into it.*

Solution to Problem 1.2.6

The statement is FALSE. The material-balance equation is a zero-dimensional model because it does not consider *spatial* and *temporal* variations of rock and fluid properties.

Problem 1.2.7

Comment on the accuracy of this statement: *Specifying constant flow rate as a boundary condition is equivalent to specifying the pressure gradient at the sandface.*

Solution to Problem 1.2.7

The statement is TRUE. A close inspection of Eq. 1.1b reveals that when flow rate q is specified at $r = r_w$, then

$$\left(\frac{\partial p}{\partial r}\right)_{r=r_w} = \frac{q\mu}{2\pi khr_w}. \tag{1.12}$$

Eq. 1.12 shows that pressure gradient at the wellbore is fixed because all the terms on the right-hand side of the equation are fixed.

Problem 1.2.8

Comment on the accuracy of this statement: *An infinite-acting assumption of classical well test analysis theory does not hold true for small reservoirs.*

Solution to Problem 1.2.8

The statement is not *necessarily* true. No matter how small the reservoir, there will be a period for a 'measurable' pressure transient to reach the boundary. Therefore, it is accurate to assume that during that period, the reservoir is infinite acting.

Nomenclature

A = cross-sectional area, L^2, ft^3
c = compressibility, Lt^2/m, psi^{-1}
h = thickness, L, ft
I = electrical current, q/t, A, and boundary conditions (external and internal)
k = permeability, L^2, darcies
K = heat conductivity, $ML/t^3/T$, Btu/hr/ft/°F
M = mathematical representation of a fluid flow problem
O = a time-series representation of the pressure surface of a production surface
p = pressure, m/Lt^2, psia
p_i = initial pressure, m/Lt^2, psia
q = production rate or flow rate, L^3/t, STB/D
q_ψ = production and/or pressure characteristics as function of time (time series)
Q = rate of heat transfer, m/t^3T, Btu/hr
r = distance from the wellbore, L, ft
r_e = radius of external boundary, L, ft
r_w = well radius, L, ft
R = electrical resistance, mL^3/tq^2, Ω
S = a forcing function applied to the system
t = time, days
β_c = transmissibility conversion factor, with a magnitude of 1.127
ΔE = voltage difference, V
ΔL = change of length, L, ft
Δp = pressure difference, m/Lt^2, psia
ΔT = temperature difference, °R
μ = viscosity, cp
ϕ = porosity, fraction
Φ = flow domain characteristics
ψ = flow process including all the thermodynamics of the reservoir system

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